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56 PINE STREET.

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THE TEMPERATURE-ENTROPY DIAGRAM

BY

CHARLES W. BERRY

Instructor in Mechanical Engineering in the Massachusetts Institute of Technology

FIRST EDITION

FIRST THOUSAND

NEW YORK

JOHN WILEY & SONS

LONDON: CHAPMAN & HALL, LIMITED

1905

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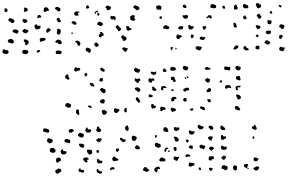

ROBERT DRUMMOND, PRINTER, NEW YORK

PREFACE.

THIS little volume was prepared for the use of students of thermodynamics, and therefore I have endeavored to bring together in logical order certain information concerning the construction, interpretation, and applications to engineering problems of the temperature-entropy diagram, which otherwise would not be readily available for them, as such information is scattered throughout various treatises. The book is not intended for the advanced student, as he is already familiar with its contents, neither is it expected that one entirely ignorant of thermodynamics can use it to advantage, as the reader is assumed to have an elementary knowledge of the fundamental theory and equations. An exhaustive treatment has not been attempted, but it is believed that the graphical presentation here given will aid the student to a clearer comprehension of the fundamental principles of thermodynamics and make it possible for him to read understandingly more pretentious works.

CHARLES W. BERRY.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
January, 1905.

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INTRODUCTION.

It seems necessary in a book dealing with the application of the temperature-entropy diagram to discuss briefly that "ghostly quantity," entropy, although I do not intend to give any new definition of a function already too variously defined, but rather to pick out such of the present ones as are correct.

One has but to plot an irreversible adiabatic process in the temperature-entropy plane to realize once and for all that the entropy does not necessarily remain constant along an adiabatic line. In fact isentropic and adiabatic changes coincide only when the latter process is reversible: and such a change practically never occurs in nature. For example, in one irreversible adiabatic expansion representing the flow through a non-conducting porous plug, the heat added is zero, so that $\int \frac{dQ}{T} = 0$, but nevertheless the entropy of the substance increases. It is even possible to imagine an irreversible process which is at the same time isentropic. Suppose a gas to expand through a nozzle

losing heat by radiation and conduction and also undergoing friction losses whereby part of its kinetic energy is dissipated and restored to the gas as heat. The loss of heat by radiation and conduction will reduce the entropy of the gas, while the gain of heat by friction will increase it. It is possible to consider these two opposing influences as equal, and then the flow will be isentropic although not adiabatic.

The entropy of a substance, just as much as its intrinsic energy, specific volume, specific pressure, or temperature, has a definite value for each position of the state point upon the characteristic surface, and the increase in the value of the entropy in changing from one point to another is a definite quantity regardless of the path chosen. The magnitude of this increase is equal to $\int_1^2 \frac{dQ}{T}$, taken along any *reversible* path between these points. This fact has led to the inexact definition of change of entropy, $d\phi = \frac{dQ}{T}$, a definition true only for ideal reversible processes and hence utterly wrong when applied to actual irreversible processes, as in general $d\phi > \frac{dQ}{T}$.

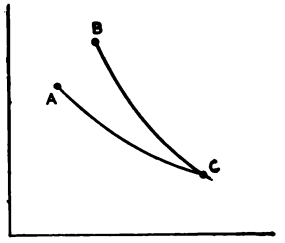
Since for reversible cycles $d\phi = \frac{dQ}{T}$, it follows that the heat added during any reversible change is equal to $Q = \int_1^2 T d\phi$, and for an isothermal process

$Q=T(\phi_2-\phi_1)$. This is undoubtedly the basis for all the physical analogies attempting to explain entropy as heat-weight, etc., and also for the name "heat diagram" applied to the temperature-entropy diagram. The area under a curve in the $T\phi$-plane is equal to the heat received from, or rejected to, some outside body only when the process is reversible.

Similarly if the specific pressure and specific volume of a gas could be ascertained at various points in its passage through a porous plug, these points if plotted would form a pv-curve giving a true history of the movement of the state point, but the area under the curve would not represent work, as no external work has been performed.

Preston in his *Theory of Heat* says: "The entropy of a body being taken arbitrarily as zero in some standard condition A, defined by some standard temperature and pressure (or volume), the entropy in any other state B is the value of $\int \frac{dQ}{T}$ taken along any reversible path by which the body may be brought to B from the standard state A. The path may obviously be an arc AC of an isothermal line passing through the point defining the standard state, together with the arc BC of the adiabatic line passing through B. The entropy in the state B may consequently be measured thus. Let the volume be changed adiabatically" (reversible process) "*until* the standard temperature T is attained,

and then change the volume isothermally until the standard pressure is attained. If the quantity of heat



imparted during the latter operation be Q, the entropy in the state B is $\phi = \frac{Q}{T}$

"In this operation the temperature and pressure are supposed uniform throughout the body. . . . If, however, any body be subject to operations which produce inequalities of temperature in the mass, there will be a transference of heat from the warmer to the colder parts by conduction and radiation, and although the body may neither receive heat from nor give it out to other bodies (so that the transformation is adiabatic throughout), yet on account of the inequalities of temperature, the entropy of the mass will increase, . . . and under these circumstances the transformation will not be isentropic."

Swinburne in his *Entropy* says: "Entropy may be defined thus: Increase of entropy is a quantity which, *when multiplied by* the lowest available temperature,

gives the incurred waste. In other words, the increase of entropy multiplied by the lowest temperature available gives the energy that either has been already irrevocably degraded into heat during the change in question, or must, at least, be degraded into heat in bringing the working substance back to the standard state. . . .

"Thus the entropy of the body in state *B* is not a function of the heat actually taken in during its change from *A* to *B*, as the change must have been partially, and may have been wholly, irreversible; but it can be measured as a function of the heat which would have to be taken in to change from *A* to *B* reversibly, or which would have to be given out if the substance were changed from *B* to *A* reversibly, which amounts to the same thing. . . .

"The entropy of a body therefore depends only on the state, and not on its past history."

Planck in his *Treatise on Thermodynamics* writes (see English translation by Ogg):

"A process which can in no way be completely reversed is termed *irreversible*, all other processes *reversible*. That a process may be irreversible, it is not sufficient that it cannot be directly reversed. This is the case with many mechanical processes which are not irreversible. The full requirement is, that it be impossible, even with the assistance of all agents in nature, to restore everywhere the exact initial state *when the process* has once taken place. . . . The gen-

eration of heat by friction, the expansion of a gas without the performance of external work, and the absorption of external heat, the conduction of heat, etc., are irreversible processes.

"Since there exists in nature no process entirely free from friction or heat-conduction, all processes which actually take place in nature, if the second law be correct, are in reality irreversible; reversible processes form only an ideal limiting case. They are, however, of considerable importance for theoretical demonstration and for application to states of equilibrium.

"*If a homogeneous body be taken through a series of states of equilibrium, that follow continuously from one another, back to its initial state, then the summation of the differential* $\frac{dE+pdv}{T}$ *extending over all the states of that process gives the value zero.* It follows that, if the process be not continued until the initial state, 1, is again reached, but be stopped at a certain state, 2, the value of the summation $\int_1^2 \frac{dE+pdv}{T}$ depends only on the states 1 and 2, not on the manner of the transformation from state 1 to state 2. . . .

"The (above) integral . . . has been called by Clausius the entropy of the body in state 2, referred to state 1 as the zero state. The entropy of a body in *a given state, like the* internal energy, is completely

determined up to an additive constant, whose value depends on the zero state.

"*It is impossible in any way to diminish the entropy of a system of bodies without thereby leaving behind changes in other bodies.* If, therefore, a system of bodies has changed its state in a physical or chemical way, without leaving any change in bodies not belonging to the system, then the entropy in the final state is greater than, or, in the limit, equal to the entropy in the initial state. The limiting case corresponds to reversible, all others to irreversible, processes.

"The restriction . . . that no changes must remain in bodies outside the system is easily dispensed with by including in the system all bodies that may be affected in any way by the process considered. The proposition then becomes:

"*Every physical or chemical process in nature takes place in such a way as to increase the sum of the entropies of all the bodies taking any part in the process. In the limit, i.e. for reversible processes, the sum of the entropies remains unchanged.* This is the most general statement of the second law of Thermodynamics."

SYMBOLS USED IN THE FOLLOWING PAGES.

$A = \frac{1}{J}$ = Heat equivalent of a unit of work.

Apu = Heat equivalent of the external work of vaporization.

c = General expression for the specific heat during any change.

c_p = Specific heat at constant pressure.

c^v = Specific heat at constant volume.

E = Internal (intrinsic) energy in work units = $S + I$.

η = Thermal efficiency of an engine.

F = Area (*Flache*).

G = Weight (*Gewicht*).

g = Acceleration due to gravity.

H = Total heat above some arbitrary zero, = q, $q + xr$, λ, $\lambda + c_p(t_s - t)$.

h = Specific heat of dry saturated vapor.

I = Internal energy due to separation of molecules.

$K = \frac{c_p}{c_v}$ for a perfect gas.

λ = Total heat of dry saturated vapor = $q + r$.

n = Exponent of the polytropic change $pv^n = c$.

p = Specific pressure.

ϕ = General expression for entropy.

Q = Heat received from or exhausted to some outside body.

q = Heat of the liquid.

$R = \frac{pv}{T}$, for a perfect gas.

r = *Total latent* heat of vaporization = $\rho + Apu$.

SYMBOLS USED IN THE FOLLOWING PAGES.

ρ = Internal latent heat of vaporization.

$\frac{r}{T}$ = Entropy of vaporization.

S = Internal energy due to vibration of molecules.

s = Specific volume of a dry saturated vapor.

σ = Specific volume of a liquid.

T = Temperature in degrees absolute, Fahrenheit or Centi grade.

t = Temperature in degrees Fahrenheit or Centigrade.

t_s = Temperature of superheated vapor.

θ = Entropy of the liquid = $\int \frac{cdt}{T}$.

u = Increase of volume due to vaporization = $s - \sigma$.

v = General value of specific volume = σ, $xu + \sigma$, s, etc.

V = Velocity in linear units per second.

$\frac{V^2}{2g}$ = Kinetic energy of a jet in work units.

W = Work performed by or upon a substance.

x = Quality of a unit weight of a mixture of a liquid and its vapor.

THE TEMPERATURE-ENTROPY DIAGRAM.

CHAPTER I.

GENERAL DISCUSSION. REVERSIBLE PROCESSES AND CYCLES.

THE condition of a substance is in general completely defined by any two of its five characteristic properties, specific pressure, specific volume, absolute temperature, intrinsic energy, and entropy. The relations existing between any three of these qualities being expressed by the formula $z=f(x, y)$. Exceptions to this occur in the case of saturated vapors, where specific pressure and temperature alone do not suffice to define completely the state of the body, and in the case of perfect gases, where isodynamic and isothermal changes coincide.

In the analytical solution of thermodynamic problems those formulæ are used which contain the properties most important for the investigation in hand. *Similarly in graphical* solutions any pair of character-

istics may be used as coordinates as convenient and then the curves, $y=f(x)$, expressing the relations between the various characteristics drawn in this xy-plane, assume different forms according to the laws of variation of the five variables p, v, t, E, and ϕ.

The pressure-volume diagram, or pv-diagram, is the one most widely used, as its coordinates are those common to every-day experience, but for some investigations of heat-transference, changes of temperature, changes of entropy, etc., the temperature-entropy diagram, or the $T\phi$-diagram, lends more ready assistance.

As a consequence of the fundamental relation $z=f(x, y)$, any curve $p=f_1(v)$ in the pv-plane has its counterpart $\phi=f_2(T)$ in the $T\phi$-plane; both being but special projections of the same change on the characteristic surface.

In the following a discussion will be given of the different forms assumed by the curve $\phi=f(t)$, in the case of perfect gases, saturated steam, and superheated steam under the conditions of constant temperature, constant entropy, constant pressure, constant volume, constant intrinsic energy, etc., and also a consideration of the interpretation to be given to the areas included between any given curve and the axis, together with certain other interesting details.

If $O\phi$ and OT (Fig. 1) represent the axis of entropy *and temperature respectively*, any isothermal change,

as at the temperature t_2, will be represented by a horizontal line *ab*, and similarly any isentropic change (or adiabatic in the case of a reversible process) will be represented by a vertical line, as *cd*. The forms of the curves for constant pressure, etc., will vary with

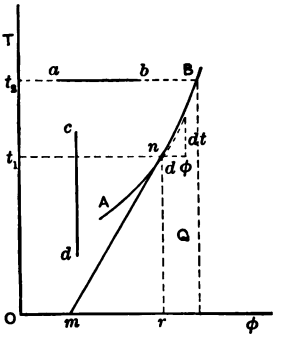


FIG. 1.

the state of the substance and will be investigated for each special case.

Let *AB* represent any *reversible* process and let *c* be the specific heat of the substance during this change. In order to increase the temperature of a unit weight of the substance by the amount *dt* there will be necessary the expenditure of the heat $dQ = cdt$. But for a reversible cycle there exists the further relation $d\phi = \frac{dQ}{T}$, or $dQ = Td\phi$, and therefore

$$\left.\begin{aligned} &dQ = cdt = Td\phi, \\ \text{or} \quad &Q = \int_{t_1}^{t_2} cdt = \int_{\phi_1}^{\phi_2} T\,d\phi. \end{aligned}\right\} \quad \ldots \quad (1)$$

Now $\int_{t_1}^{t_2} T d\phi$ represents the area included between the curve AB and its projection upon the ϕ-axis. Hence *the heat necessary to produce any reversible change is represented by the area under the curve in the $T\phi$-plane.*

The differential form of equation (1) leads at once to the expression

$$c = T\frac{d\phi}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

At any point n of the curve AB draw the tangent nm and construct the infinitesimal triangle $dt\,d\phi$. Then from similar triangles

$$mr : T = d\phi : dt,$$

or

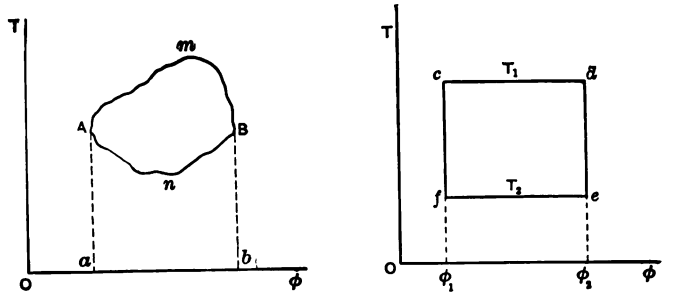
$$mr = T\frac{d\phi}{dt} = c;$$

i.e., if from any point in a curve representing a reversible process a tangent be drawn, the length of the subtangent on the ϕ-axis represents the momentary value of the specific heat for that change.

In Fig. 2 (A) let $AmBnA$ represent a reversible cycle in which AmB is the forward stroke and the area $aAmBb$ represents the heat received from external sources, and where BnA is the return stroke and the area $aAnBb$ represents the heat rejected. On the completion of the cycle the intrinsic energy has regained its initial value and therefore the difference between

the heat received and the heat rejected, i.e., the magnitude of the enclosed area $AmBnA$, must represent the amount of heat changed into work.

Carnot Cycle.—In the case of the Carnot engine this choice of coordinates leads to a beautiful simplicity. The cycle Fig. 2 (*B*) becomes a rectangle consisting



(*A*) FIG. 2. (*B*)

of (1) the isothermal expansion *cd*, during which is received the heat $Q_1 = \int_{\phi_1}^{\phi_2} T_1 \, d\phi = T_1(\phi_2 - \phi_1)$, represented by the total rectangle $\phi_1 cd\phi_2$; (2) the adiabatic line *de*; (3) the isothermal compression during which is rejected the heat $Q_2 = \int_{\phi_1}^{\phi_2} T_2 d\phi = T_2(\phi_2 - \phi_1)$, represented by $\phi_1 fe\phi_2$; and (4) the adiabatic compression *fc*. The heat changed into work is $Q_1 - Q_2 = T_1(\phi_2 - \phi_1) - T_2(\phi_2 - \phi_1) = (T_1 - T_2)(\phi_2 - \phi_1)$. The efficiency, $\eta = \frac{Q_1 - Q_2}{Q_1}$, is simply the ratio of the rectangles *cdef* and $\phi_1 cd\phi_2$, and as these have the same base, the

areas are proportional to the altitudes and at once the efficiency becomes $\eta = \frac{T_1 - T_2}{T_1}$.

Isodiabatic Cycles. — In deducing the equation $d\phi = \frac{dQ}{T}$ for reversible cycles, use is made of the Carnot cycle and it is shown that no other cycle can have a greater efficiency. There are, however, a number of other cycles having the same efficiency as the Carnot within the same range of temperature.

In Fig. 3, let the reversible cycle *abcd* be formed by

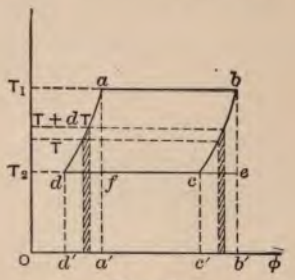


FIG. 3.

the two isothermals *ab* and *cd* and of the two curves *bc* and *da*. The curve *bc* is arbitrary, but *da* is drawn like it, being simply displaced to the left. The heat turned into work during the cycle is *abcd*, or equal to that of the equivalent Carnot cycle *abef*, but the heat supplied is $d'daa' + a'abb'$, or is greater than that needed to perform the same work with the Carnot cycle by *the amount* $d'daa'$. Now the heat rejected from *b* to *d* *consists of the two* parts, $cdd'c'$, which passes to the

refrigerator at the lowest temperature T_2 and is of necessity lost, and *bb'c'c*, which is rejected during a dropping temperature and which is strictly equal to the heat required to carry out the reverse operation *da*. Instead of being wasted, the heat rejected along *bc* might be stored up and returned to the fluid along *da*, thus making no further demands upon the source of heat. In this manner the one operation would balance the other and the heat actually required from an external source would be represented by *a'abb'*, as in the Carnot cycle. The heat actually rejected would also be represented by *d'dcc'*, equal to *a'feb'*, as in the Carnot cycle. Thus the *adiabatic* lines of the Carnot cycle would be replaced by two lines *bc* and *da*, along which the interchanges of heat are compensated. The efficiency of such *isodiabatic* cycles would thus be equal to that of the Carnot cycle.

The operations along the lines *bc* and *da* may be imagined as follows. Heat only flows spontaneously from one body to another at a lower temperature. Thus the heat given up along *bc* can be stored and utilized to effect the operation *da*, if the process be subdivided into very small differences of temperature and each portion of the heat rejected at the momentary temperature. Thus the heat rejected along *bc* between the temperatures $T+dT$ and T (Fig. 3) is returned along a portion of *da* at the same temperature.

The difficulty is to find a practical regenerator to

perform this duty. It must have large heat-conducting surfaces, and consist of a number of subdivisions at all temperatures between T_1 and T_2, and there must be no conduction of heat from any part to the next at lower temperature. To accomplish the operation along *bc*, the working fluid passes successively through these divisions and deposits in each a part of its heat. During the process *da* the fluid passes again through the divisions but in a reverse direction, from the coldest to the hottest. This ideal process can only be roughly realized as the regenerator has a limited conductibility, permits the flow of heat between adjacent sections, and can only store up heat if the fluid is much hotter and refund it if the fluid is much colder than itself. Therefore only the upper portion of the line *bc* can actually be utilized to refund heat gratuitously along the lower portion of *da*. Hence in practice it should be possible to obtain a higher efficiency with an engine working on the Carnot cycle, because those working on the isodiabatic cycle have the added losses of the regenerator. But theoretical efficiency is only one of several factors entering into the design of an actual engine and may be more than balanced by the increased size, cost, strength, etc., required to attain it.

Reversible Cycles Less Efficient than the Carnot Cycle.—In Fig. 4 let *AmB* represent that part of a reversible cycle during which the maximum temperature t_1 is reached and the heat *aAmBb* is received from external

sources, and let *BnA* represent the return stroke during which the heat rejected is equal to *aAnBb* and the lowest temperature equal to t_2. A Carnot cycle for the same limits of temperature would give the efficiency $\eta = \frac{uqrs}{aqrb}$. Any deviation of the expansion line *AmB* from the isothermal *qmr* diminishes both the numerator and denominator of η by the same quantity, viz., the

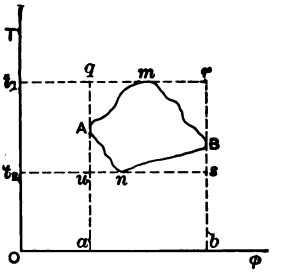


FIG. 4.

areas *Aqm* and *mrB*, and hence the efficiency will be less. Further, any deviation of the compression line *BnA* from the lower isothermal *snu* will descrease the numerator and increase the denominator of η, thus producing a still further decrease in the efficiency. *Therefore in working between any two temperatures the highest possible efficiency will not be attained unless all the heat received is taken in at the upper temperature, and all the heat rejected is given out at the lower temperature.* The only exception to this is in the case of the isodiabatic cycles already considered.

CHAPTER II.

THE TEMPERATURE-ENTROPY DIAGRAM FOR PERFECT GASES.

THE fundamental heat equations for a perfect gas are

$$\left.\begin{aligned} dQ &= c_v dt + (c_p - c_v) T\frac{dv}{v}, \\ dQ &= c_p dt - (c_p - c_v) T\frac{dp}{p}, \\ dQ &= c_v T\frac{dp}{p} + c_p T\frac{dv}{v}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

and

$$c_p - c_v = AR, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which for reversible processes give three different expressions for entropy:

$$\left.\begin{aligned} \phi_2 - \phi_1 &= c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{v_2}{v_1}, \\ \phi_2 - \phi_1 &= c_p \log_e \frac{T_2}{T_1} - (c_p - c_v) \log_e \frac{p_2}{p_1}, \\ \phi_2 - \phi_1 &= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}. \end{aligned}\right\} \quad . \quad . \quad (5)$$

The curves for constant volume and constant pressure, respectively, in the $T\phi$-diagram become

$$\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and $$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If $p=f(v)$ be the pv-projection of any curve on the characteristic surface $pv=RT$; to find the $T\phi$-projection of the same curve it is only necessary to find $\frac{v_2}{v_1}=f_1(t)$ or $\frac{p_2}{p_1}=f_2(t)$ from the above equations and to substitute them in the first and second forms, respectively, of equations (5).

The most general form of $p=f(v)$ with which the engineer is concerned is that of an indicator card, namely,

$$pv^n = p_1 v_1^n = \text{a constant}, \quad . \quad . \quad . \quad . \quad (8)$$

where the exponent is determined from any two points by means of the formula

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1}.$$

The respective projections of equation (8) on the Tv- and Tp-planes, as found by combination with the characteristic equation, are

$$T_1 v_1^{n-1} = T_2 v_2^{n-1} = \text{a constant} \quad . \quad . \quad . \quad (9)$$

and $$T_1 p_1^{\frac{1-n}{n}} = T_2 p_2^{\frac{1-n}{n}} = \text{a constant}, \quad . \quad . \quad (10)$$

whence it follows that

$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{T_1}{T_2}\right)^{\frac{n}{1-n}} \quad . \quad . \quad . \quad (11)$$

The substitution of equations (11) in the corresponding forms of equations (5) gives

$$\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$= \left(c_v - \frac{c_p - c_v}{n-1}\right) \log_e \frac{T_2}{T_1} = c_v \cdot \frac{n-K}{n-1} \cdot \log \frac{T_2}{T_1}, \quad . \quad (12)$$

or

$$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1} - (c_p - c_v) \log_e \left(\frac{T_1}{T_2}\right)^{\frac{n}{1-n}}$$

$$= \left(c_p - \frac{n(c_p - c_v)}{n-1}\right) \log_e \frac{T_2}{T_1} = c_v \cdot \frac{n-K}{n-1} \cdot \log_e \frac{T_2}{T_1}. \quad (12)$$

Equations (8), (9), (10), and (12) represent simply different projections of the same curve on the characteristic surface upon different planes.

From equation (2), for perfect gases,

$$d\phi = c \frac{dt}{T}, \quad \text{or} \quad \phi_2 - \phi_1 = c \log_e \frac{T_2}{T_1}. \quad . \quad . \quad (13)$$

Comparison of equations (12) and (13) shows that the specific heat for the general expansion $p_1 v_1^n = p_2 v_2^n$ is

$$c = c_v \frac{n-K}{n-1}; \quad . \quad . \quad . \quad . \quad . \quad (14)$$

i.e., *the specific heat of any expansion of a perfect gas, $pv^n = \text{constant}$, can be expressed as a function of the specific heat at constant volume, the ratio of c_p and c_v, and the exponent n of the expansion.*

Let us consider equations (8), (12), and (14) for the following special cases:

(1) For $T=\text{const.}$, $pv=RT=\text{constant}$, and $n=1$.

Equation (14) becomes $c=c_v\cdot\frac{1-K}{1-1}=\infty$; i.e., for an isothermal change the heat capacity of the substance is infinite; that is to say, no matter how much heat is added the temperature will not change.

Equation (12) becomes $\phi_2-\phi_1=\infty\cdot 0$; that is, the value of ϕ may undergo any change whatever and the $T\phi$-curve simply becomes $T=\text{const.}$

(2) For $\phi=\text{constant}$, equation (12) gives $n=K$, and equation (14) becomes $c=c_v\frac{K-K}{K-1}=0$; i.e., for an isentropic change the heat capacity of the substance is *nil;* that is to say, the temperature of the substance can be increased or diminished without the addition or subtraction of heat *as heat.* Equation (8) becomes $pv^K=\text{constant}$.

(3) For $p=\text{constant}$, equation (8) becomes $v_1^n=v_2^n$, hence n equals zero.

From (14) $c=c_v\frac{0-K}{0-1}=c_v\cdot\frac{c_p}{c_v}=c_p$, and equation (12) becomes $\phi_2-\phi_1=c_p\log_e\frac{T_2}{T_1}$, as already found in equation (7).

(4) For $v=\text{constant}$, the only value of n which will satisfy $p_1v_1^n=p_2v_1^n$ is $n=\infty$; so that equation (8) becomes $v=\text{constant}$.

Equation (14) gives $c = c_v \dfrac{\infty - K}{\infty - 1} = c_v$, whence from equation (12) $\phi_2 - \phi_1 = c_v \log_e \dfrac{T_2}{T_1}$, as previously found in equation (6).

We are now in a position to transfer any curve from the pv-plane to the $T\phi$-plane as soon as we know the value of c_v for the given gas.

In Fig. 5 let *abcd* represent a cycle consisting of two

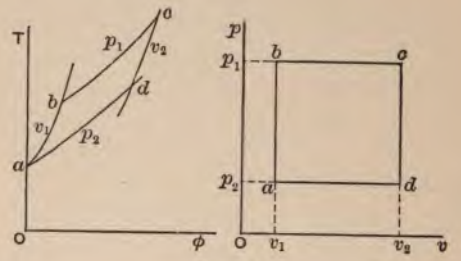


FIG. 5.

curves of constant volume and two of constant pressure indicated by a rectangle in the pv-plane. In the $T\phi$-plane start with the value of the entropy at a as the zero-point. The curve ab will be of the nature shown, becoming steeper as T increases because the subtangent at any point represents the value of c_v and this is a constant for perfect gases. Arriving at b, the curve of constant pressure will assume some such position as *shown.* bc will not be as steep as ab at the point of *intersection because* $c_p > c_v$, i.e. the subtangent of bc

is greater than the subtangent of *ab*. Two similar curves *cd* and *da* complete the cycle.

All possible variations of the curves $pv^n = \text{constant}$, or $\phi_2 - \phi_1 = c_v \frac{n-K}{n-1} \log_e \frac{T_2}{T_1}$, are summarized in the following table and diagrams:

pv-coordinates.		*Tφ*-coordinates.	
n	Form of the Curve.	$c_v \cdot \frac{n-K}{n-1}$	Form of the Curve.
0	p = constant	c_p	T and ϕ increase or decrease together
$0<n<1$	pv^n = constant	$>c_p$	
1	pv = constant (isothermal)	∞	T = constant *
$1<n<K$	pv^n = constant	Negative	T increasing and ϕ decr'sing; T decreasing and ϕ incr'sing
K	pv^K = constant (isentropic)	0	ϕ = constant
$K<n<\infty$	pv^n = constant	$<c_v$	T and ϕ increase or decrease together
∞	v = constant	c_v	

* During an isothermal change of a perfect gas all the heat added performs external work, hence in the diagrams the isodynamic lines are superimposed upon the isothermals.

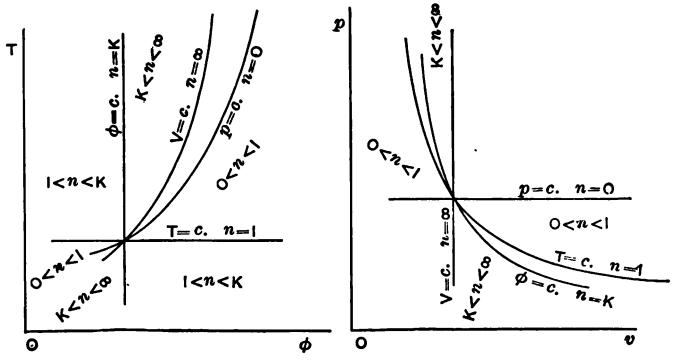


FIG. 6.

CHAPTER III.

THE TEMPERATURE-ENTROPY DIAGRAM FOR SATURATED STEAM.

Due to the very slight variations in the volume of water with increasing temperature the heat equivalent of the external work is very small, and the difference between the work performed under atmospheric pressure and that which would be performed under a pressure increasing with the temperature according to Regnault's pressure-temperature curve is negligible compared with the heat required to increase the temperature. Hence the value of the specific heat $c=f(t)$ as determined by Rowland is taken as equivalent to that of the actual specific heat required for the transformation occurring when feed-water is heated in a boiler. Similar statements hold for other fluids.

Therefore the heat required to increase the temperature of a pound of liquid by the amount dt while the pressure increases by dp is taken as

$$dq=cdt, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

whence $$q_1 = \int_{32°}^{t_1} c\,dt \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and $$\theta_2 - \theta_1 = \int_{t_1}^{t_2} \frac{c\,dt}{T}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

To make the temperature-entropy chart conform to the tables for steam and other saturated vapors, we will plot the increase of entropy and intrinsic energy and the heat added above that of the liquid at freezing-point. Hence for water the zero of the entropy scale will be at 32° F. Furthermore the chart will be constructed for one pound of water.

Point *a* in Fig. 7 represents the position of a pound of water at 32° F. on the $T\phi$-chart. Starting from here and plotting values of θ and T from the equation

$$\theta = \int_{32°}^{t} \frac{c\,dt}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

as given in the steam-tables, we obtain the "water-line" *ab*. At any temperature t the value of the specific heat c is shown by the subtangent *gf*. Further, as we proved in the general case of equation (1), the area under the curve *aefO* represents the heat of the liquid, q; that is, the number of heat-units required to warm up the water from 32° F. to the temperature t. This "water-line" is the same kind of a curve as that represented by equation (12) for perfect gases, except that

the specific heat of a gas is a constant, while that of water is a function of the temperature.

If at t the water has reached the temperature corresponding to the boiler pressure any further increase of heat will cause the water to vaporize under constant pressure and thus there will be an increase of entropy at constant temperature. This will be represented on the chart by the horizontal line *ed*, and will continue until all the water is vaporized and a

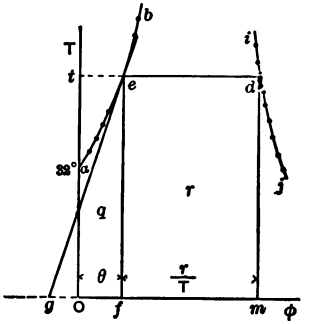


FIG. 7.

condition of dry-saturated steam reached. During this change the increase of entropy will be

$$\phi_2 - \phi_1 = \int_{\phi_1}^{\phi_2} \frac{dQ}{T} = \frac{1}{T}\int_{\phi_1}^{\phi_2} dQ = \frac{r}{T}; \quad . \quad . \quad (19)$$

that is, the length of the line *ed* may be found by taking the latent heat of vaporization r and dividing by the *absolute temperature* corresponding to t.

The area under the curve *ed* represents the heat added during vaporization or r.

The area under the curve *aed* therefore represents all the heat necessary to be added to a pound of water at 32° F. to change it in a boiler into dry steam at the temperature t, or area $Oaedm = \lambda = q + r$.

Similarly the location of the "dry-steam" point may be found for any number of temperatures, thus giving the location of the dry-steam line or saturation curve.

In the area between the water-line and the dry-steam line *ij*, or the region of vapor in contact with its liquid, lie all the values given in the steam-tables; to the right of the dry-steam curve lies the region of superheated steam.

Having given the chart with the "water-line" and "dry-steam" line located, and knowing the temperature t of the steam, it is simply necessary to draw the horizontal line *ed* and drop the two perpendiculars *ef* and *dm* and the tangent *eg*; then the diagram gives at once

$$q,\ r,\ \lambda,\ \theta,\ \frac{r}{T},\ T, \text{ and } c.$$

Let *e* in Fig. 8 represent the state point of a pound of water in a boiler under the pressure p corresponding to the temperature t. As heat is applied part of the water vaporizes and the state point moves toward *d*. At any point as *M*, the area under *eM* represents the *heat already added*, while the remaining part, under Md,

represents the heat, which must be added to complete the vaporization. That is, for the complete change from water to dry steam the state point travels the distance *ed*, and to vaporize one half it would travel one half the distance, etc. Then if *M* represents the momentary position of the state point, the ratio $eM \div ed$ will represent the fractional part of the water already vaporized;

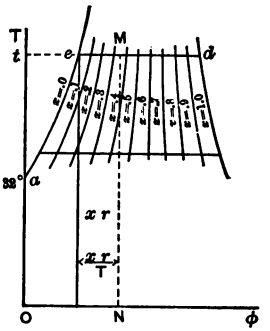


FIG. 8.

that is, the dryness of the mixture, x, is given by

$$x = \frac{eM}{ed}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Hence if the line *ed* is divided into any convenient number of equal parts (say 10) the value x can be read directly from the chart as soon as the position of the state point *M* is known. In Fig. 8, for example, $x_M = 0.35$.

Furthermore, the value of the entropy at M is represented by that of the liquid at e and the xth part of the entropy of vaporization, or

$$\phi_M = \theta_M + \frac{x_M r_M}{T_M};$$

and the total heat at M equals the heat of the liquid plus the xth part of the heat of vaporization, or

$$H_M = q_M + x_M r_M,$$

and is represented by the area $OaeMN$.

In a similar way the distance $\frac{r}{T}$ for several temperatures can be divided into the same number of equal parts, and then if all these corresponding points are connected by smooth curves, each curve will represent a change during which the fractional part of the water vaporized is constant. These are known as the x-curves.

In place of having separate scales of pressure and temperature for the ordinates of the $T\phi$-diagram, it is often convenient to take the values of p and t from the steam-tables and to plot Regnault's pressure-temperature curve in the second quadrant, as shown in Fig. 9.

Then given any pressure p_1 the corresponding temperature t_1 may be found as indicated or vice versa.

Let p_1 be the pressure and x_1 the dryness fraction of a pound of mixture, then on the chart its condi-

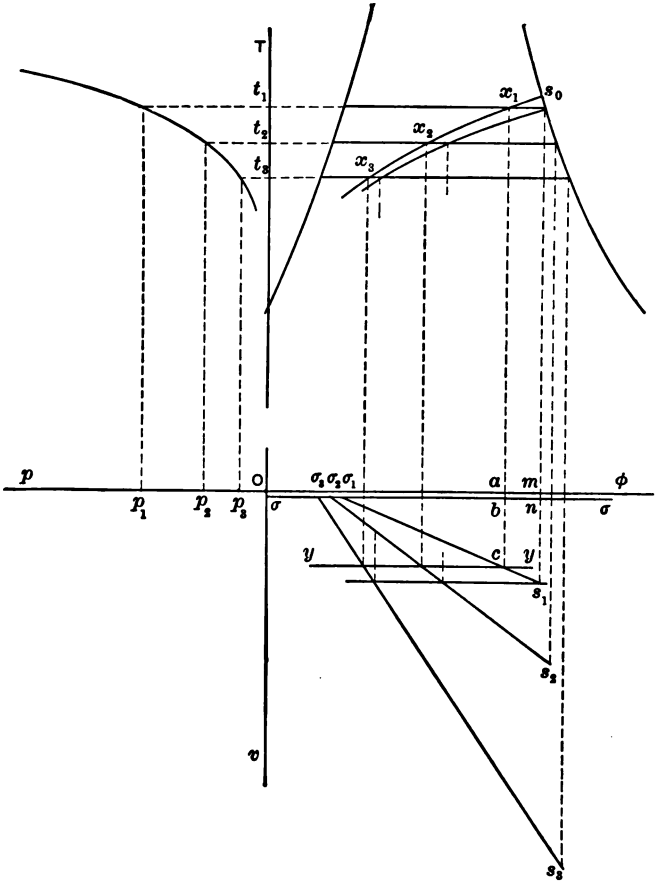


FIG. 9.

tion will be indicated by the point 1. Its volume may *be found from the steam*-tables by use of the formula $v = xu + \sigma$,

To find the location of the state point 3 at any other pressure p_3, so that $v_3 = v_1$, proceed as follows:

Draw the axis of volume opposite to that of temperature and lay off along Ov the distance $O\sigma$ equal to the volume of one pound of water. The variations of σ with t may be neglected and σ set equal to 0.016 cubic feet. Draw $\sigma\sigma$. Project θ_1 and $\theta_1 + \frac{r_1}{T_1}$ on the ϕ-axis, and from the latter point lay off the distance ms_1 equal to the volume of one pound of saturated steam as given in the steam-table for $t_1°$. Prolong the perpendicular from θ_1 until it intersects $\sigma\sigma$ at σ_1. Connect this last point with s_1. This line $\sigma_1 s_1$ shows the increase of the volume and the entropy during vaporization. Project x_1 upon $O\phi$ and continue until it intersects the $v\phi$-curve at c. Then $bc = x_1 u_1$ and $ac = x_1 u_1 + \sigma_1 = v_1$. Through c draw yy parallel to $O\phi$. For any pressure p_3 draw the corresponding $v\phi$-curve $\sigma_3 s_3$, and where this intersects yy the volume will be $v_3 = v_1 = x_3 u_3 + \sigma$. Projected up this intersects the isothermal t_3 in 3, giving the desired dryness fraction x_3. Points 1 and 3 have the same volume v_1. Other points, as 2, etc., may be found and connected with a smooth curve. This will intersect the dry-steam line at some point $s_0 = v_1$. In this manner similar constant volume curves can be constructed to cover the entire diagram.

Suppose it is required to find the heat necessary

to cause a change from some point L to an adjacent point $L+dL$. (See Fig. 10.)

$$\phi=\theta+\frac{xr}{T}=\int\frac{cdt}{T}+\frac{xr}{T},$$

$$d\phi=\frac{cdt}{T}+\frac{x}{T}dr+\frac{r}{T}dx-\frac{xr}{T^2}dt,$$

$$dQ=cdt+xdr+rdx-\frac{xr}{T}dt+(cxdt-cxdt)$$

$$=c(1-x)dt+rdx+x\left(c-\frac{r}{T}+\frac{dr}{dt}\right)dt. \quad . \quad . \quad (21)$$

To interpret the last term imagine this change to occur along the dry-steam line. Then $x=1$ and $dx=0$, whence

$$dQ=\left(c-\frac{r}{T}+\frac{dr}{dt}\right)dt(=Td\phi) \quad . \quad . \quad . \quad (21a)$$

The comparison of this with equation (2) shows that $c-\frac{r}{T}+\frac{dr}{dt}$ is the "specific heat" of dry-saturated steam,

or
$$h=c-\frac{r}{T}+\frac{dr}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The diagram shows at once that h is negative, i.e. to move along the dry-steam line with increasing temperature heat must be rejected.

Ether has a positive value for h. This signifies that the saturated-vapor line for ether slants to the right *instead of to the left.*

Hirn found that steam condensed upon adiabatic expansion and Cazin that it did not condense upon compression. The reason for this is at once clear from Fig. 10. Expanding from *a* the reversible adiabatic

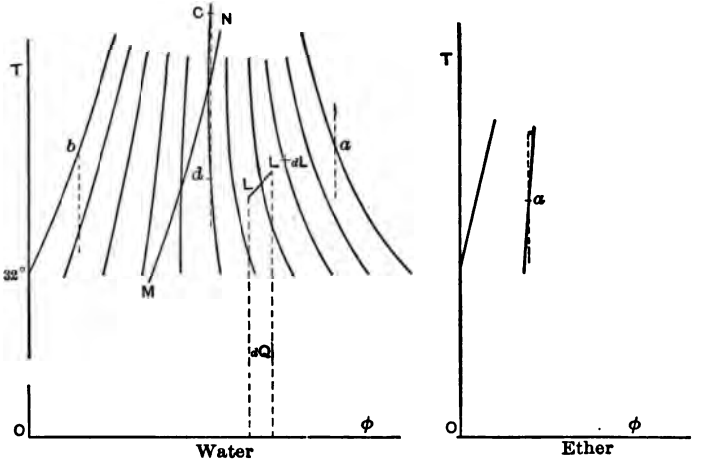


FIG. 10.

line for water cuts successively "x-lines" of decreasing value, showing condensation. Compressed adiabatically from *a* the steam would at once become superheated. Exactly the reverse occurs with ether, condensation occurring during adiabatic compression, superheating during expansion.

If water is permitted to expand adiabatically from *b* it is partially vaporized, as shown by the adiabatic line cutting increasing values of x. Similarly if very *wet steam is* compressed it condenses.

An inspection of the x-curves near the value $x=0.5$ shows that they change from convex to concave and that it is thus possible with water for the reversible adiabatic to cut an x-curve twice at different temperatures, as at c and d; i.e., it is possible at the end of an isentropic expansion to have the same value of x as at the beginning. Thus

$$\phi_1=\phi_2=\theta_1+\frac{x_1r_1}{T_1}=\theta_2+\frac{x_1r_2}{T_2}, \quad \text{or} \quad x_1=\frac{\theta_1-\theta_2}{\frac{r_2}{T_2}-\frac{r_1}{T_1}}. \tag{23}$$

Consequently there must exist some adiabatic which is tangent to this x-curve. Above the point of tangency the x-curve slants to the right and possesses a positive specific heat, below it the curve slants to the left and has a negative specific heat. The values of the specific heat above and below the point of tangency diminish in magnitude as the tangent is approached and at the point of tangency are identical and equal to zero; that is, just there the temperature may be raised without the addition of heat because the change is isentropic.

If these points of zero specific heat are determined for all the x-curves and connected by the smooth curve MN, the chart is divided into two portions, such that to the left of MN the specific heats along the x-lines are *positive*, i.e. isentropic expansion will be accompanied *by vaporization, and to* the right of MN the specific

heats are negative and isentropic expansion will be accompanied by condensation. The curve MN is known as the zero-curve.

So far we have located p, v, t, ϕ, and x, and to make the definition of the point complete it is only necessary to draw a curve of constant intrinsic energy. This could be done by solving $E_1 = E_2 = E_3 = \text{etc.}, = q_1 + x_1\rho_1 = q_2 + x_2\rho_2 = q_3 + x_3\rho_3 = \text{etc.}$, for the proper values of x and connecting these by a smooth curve. This would be very laborious, as there does not seem to be any convenient graphical construction. Fortunately it is possible not to draw the isodynamic curves, but to find an area which represents the value of the intrinsic energy for any state point and at the same time to divide r into two areas proportional to ρ and Apu respectively.

The first fundamental heat equation

$$dQ = \left(\frac{dQ}{dt}\right)_v dt + \left(\frac{dQ}{dv}\right)_t dv$$

becomes, when made to conform to the limitations of the first and second laws of thermodynamics,

$$dQ = c_v dt + AT\left(\frac{dp}{dt}\right)_v dv.$$

This is applicable to the case of saturated vapors because the state point is uniquely defined by the *intersection of* the temperature and volume curves.

Equating the coefficient of the last terms,

$$\left(\frac{dQ}{dv}\right)_t = AT\left(\frac{dp}{dt}\right)_v,$$

a relationship which holds good for any reversible change. During the process of evaporation t is a constant and

$$\left(\frac{dQ}{dv}\right)_t = \frac{r}{s-\sigma} \quad \text{or} \quad \frac{r}{u} = AT\frac{dp}{dt}, \quad . \quad . \quad (24)$$

whence

$$Apu = \frac{r}{T}\cdot\frac{p}{\frac{dp}{dt}} = \frac{r}{T}\frac{1}{\frac{1}{p}\frac{dp}{dt}} \quad . \quad . \quad . \quad . \quad (24a)$$

Let a (Fig. 11) be the point of which the intrinsic energy is to be obtained, p_a is the corresponding pressure. From a' on Regnault's tp-curve draw the tangent $a'b'$. From similar triangles it is evident that the subtangent equals $p\cdot\frac{dt}{dp}$. Draw $b'b$ parallel to $a'a$. Then the rectangle $abcd$ has the area $p\cdot\frac{dt}{dp}\times\frac{r}{T} = Apu$ (see eq. 24a).

That is, $abcd$ represents the external heat of vaporization and the rest of r, namely $bckf$, must equal ρ. Hence the intrinsic energy of the point a is given by

$$E_a = OgdkO + kcbf = \mathfrak{q} + \mathfrak{p}.$$

If M represents the state point

$$dMnc = x_M Apu,$$

$$cnmk = x_M \rho,$$

and

$$E_M = q + x_M \rho.$$

If for each point a of the dry-steam line a corresponding point c, which divides the absolute temperature

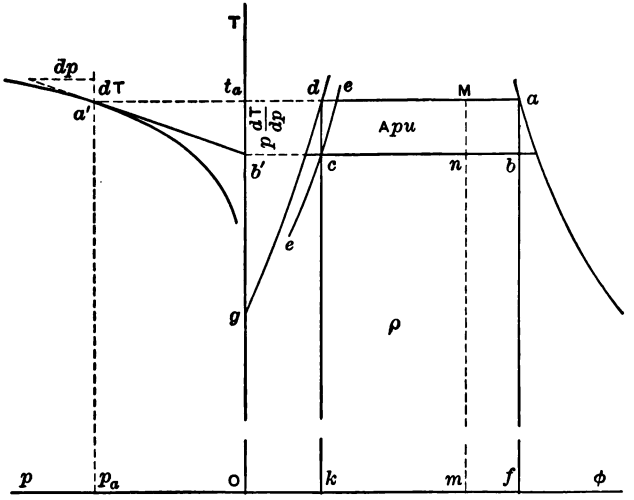


FIG. 11.

into two parts proportional to Apu and ρ, is determined the curve *ee* will be located. The curve *ee* being located once for all, the intrinsic energy at any point M can be found by drawing the isothermal Md and the adiabatics dk and Mm. At c, the point of intersection of dk with *ee*, draw the isothermal cn. Area $OgdcnmO$ gives the desired value.

The temperature-entropy diagram for steam thus enables one to find directly the following quantities: p, t, v, E, ϕ, x, c, h, s, q, r, λ, ρ, θ, $\frac{r}{T}$, and Apu. That is, the diagram is equivalent to a set of steam-tables and in some ways superior to them in that it enables one to obtain a comprehensive idea of the changes taking place between these quantities.

Let the curve MN in Fig. 12 represent the $T\phi$-pro-

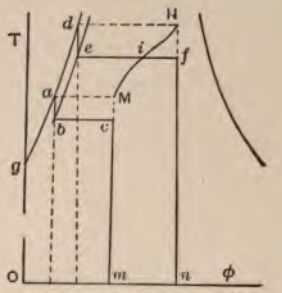


FIG. 12.

jection of some reversible change. During the change MN there has been added from some external source the amount of heat $MNnm$. At the same time the intrinsic energy has increased from $OgabcmO$ to $OgdefnO$, the increase being shown by the area $adefnmcba$. The area $MifnmM$ is common to both the heat added and the increase of intrinsic energy, and as the remaining part of the intrinsic energy increase, $adeiMcba$, is greater than the remaining portion of the heat added, $iNfi$, it follows that the heat added does

not equal the increase of internal energy and hence an amount of work must have been performed upon the substance equal to the difference in area of these two surfaces; i.e., the external work performed upon the substance equals $adeiMcba - iMfi$. These areas may be found readily with a planimeter.

The performance of external work upon the substance might have been foretold at once from the diagram, because the volume of N is less and the pressure greater than that for M.

Fig. 13 represents another type of reversible change. During the transformation MN, there is added the

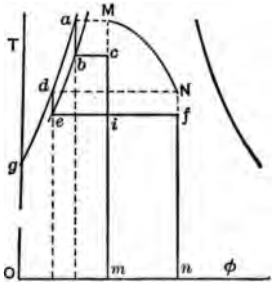


FIG. 13.

heat $MNnm$, and the intrinsic energy changes from $OgabcmO$ at M to $OgdefnO$ at N. The portion $OgdeimO$ is common to both, and the intrinsic energy at N will be greater or less than it was at M according as the area $ifnmi$ is greater or less than the area $abcieda$. The magnitude and sign of this difference may be determined each time by the use of planimeters.

In Fig. 13 $E_n > E_m$; hence this increase of internal energy must have come from the heat added, and subtracting this difference from the total heat added will give the amount remaining for external work. This is positive in the case shown, as was to be expected, as the pressure has fallen and the volume increased.

In case there was a decrease in the internal energy that area would need to be added to the heat area in order to obtain the external work performed.

Having learned to construct and interpret the $T\phi$-diagram for saturated vapors we must now resume once more the main object of our investigation, namely, to find the location in the $T\phi$-plane of any curve given in the pv-plane or *vice versa*, so that we may eventually be able to investigate the action of a steam-engine from both its indicator and its temperature-entropy diagrams.

In the case of perfect gases it was possible to use one of the fundamental heat equations and thus obtain a simple analytical expression which could be easily plotted; for saturated steam, however, this process is too cumbersome to be of any service. Fortunately the graphical method offers a solution both simple and elegant.

In constructing the $T\phi$-diagram we have already made use of the first, second, and fourth quadrants to express $T\phi$, pt, and $v\phi$ variations respectively, and now we have but to take the third or pv-quadrant and the *diagram is complete.*

The saturation curve, or curve of constant steam-weight *ab* in the *pv*-plane, is depicted by the dry-steam curve $a'b'$ in the $T\phi$-plane, Fig. 14. The method of obtaining corresponding points aa' and bb' is shown by the projection through the point a_1 and b_1 of the *pT*-curve. *Aa* and *Bb* show the increase in volume of a pound of H_2O in vaporizing under the constant pressure p_a and p_b respectively. This same increase of volume is represented in the ϕv-plane by the curves $A''a''$ and $B''b''$ respectively. Now if only part of the pound is vaporized the actual volume will be indicated by the points, say, x_a and x_b. By projection into the ϕv-plane x_a'' and x_b'' are found. Another projection and we obtain x_a' and x_b' as the state points in the $T\phi$-plane corresponding to the points x_a and x_b in the *pv*-plane.

Suppose the pressure and volume of a pound of steam have been determined for some particular part of the stroke by means of the indicator-card and steam measurements. Let x_a represent such a point. It is now desired to find the corresponding state point in the $T\phi$-plane. The procedure is as follows:

Draw through x_a the curve of constant pressure *Aa* and determine by projection its location $A'a'$ and $A''a''$ in the $T\phi$- and ϕv-planes respectively. Then project x_a to x_a'' and finally x_a'' to x_a' and the desired determination is finished.

As a further problem suppose it is desired to locate

on the $T\phi$-plane the curve of constant volume $x_a x_c$. The point x_a is already located at x_a'. To locate x_c draw Bb and find its projections at $B'b'$ and $B''b''$.

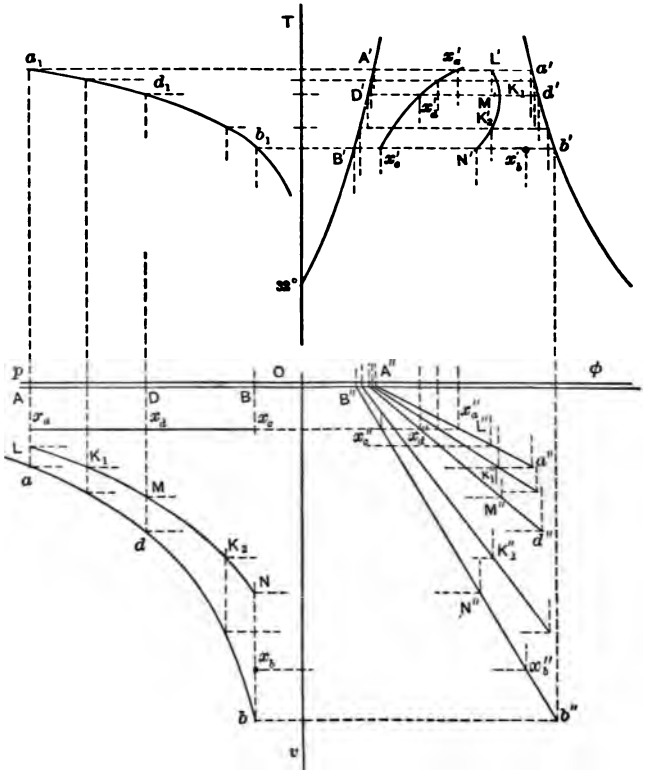


FIG. 14.

Then project x_c to x_c'' and finally to x_c'. The two end-points of the curve being determined, any intermediate point as x_d will be located in the same manner as shown. Thus after a sufficient number of points

are located, the curve of constant volume $x_a'x_c'$ may be drawn. Naturally if the chart is already provided with constant-volume curves this construction would be unnecessary.

Passing now to the most general problem consider the curve *LMN* and suppose its equation in the pv-plane to be $pv^n = p_1 v_1^n$. This would correspond to the expansion-line of an indicator-card. It is desired to find the projection of this curve in the $T\phi$-plane. The problem resolves simply into the location of a sufficient number of state points, through which a smooth curve is finally to be drawn. To locate *L*, *M*, and *N* project them on to the corresponding volume curves $A''a''$, $D''d''$, and $B''b''$ of the ϕv-plane at L'', M'', and N'', and then finally project to L', M', and N'. To properly determine the curve some intermediate point as *K* may be necessary.

CHAPTER IV.

THE TEMPERATURE-ENTROPY DIAGRAM FOR SUPERHEATED VAPORS.

In the present state of uncertainty regarding the value of the specific heat of superheated steam both at constant pressure and at constant volume, it is impossible except with a fair degree of approximation to define the location of the state point in the $T\phi$-plane.

The curves of constant pressure and constant volume are here denoted, as in the case of perfect gases, by

$$\phi_2-\phi_1=\int_{T_1}^{T_2}\frac{c_p dt}{T}, \text{ and } \phi_2-\phi_1=\int_{T_1}^{T_2}\frac{c_v dt}{T}, \text{ respectively.}$$

When the variations of c_p and c_v with the temperature are known these curves can be plotted accurately, but until then c_p and c_v may be assumed to hold constant through considerable ranges of temperature; c_p gradually increasing with higher temperatures.

As $c_p > c_v$ the constant-pressure curves will not be as steep as those of constant volume. To locate curves *of constant intrinsic* energy, there is no graphical method,

but assuming the approximate formula for the change of intrinsic energy to be true, viz.,

$$E_2 - E_1 = \frac{p_2 v_2 - p_1 v_1}{k-1},$$

it follows that the product pv is a constant along isodynamic lines. Hence starting with any point in the dry-steam curve having the pressure p and the volume s, it is simply necessary to pick out the intersection of those curves of constant pressure and constant volume giving the product $p_x v_x = ps$ and then to connect them by a smooth curve.

There would thus be five sets of curves, one set each for constant pressure, volume, temperature, intrinsic energy, and entropy. As the intersection of any two of these gives a unique location of the state point, it follows that all five characteristics may be read directly from the chart as soon as any two are known.

There is at present no simple and accurate equation $\phi = f(T)$ to represent in the $T\phi$-plane the projection of the curve $pv^n = p_1 v_1^n = \text{constant}$, so that recourse must again be had to the graphical method. Determine the pressure and volume of a sufficient number of points on the pv-curve and then plot these values on the corresponding pressure and volume curves in the $T\phi$-plane. The curve thus produced will be an approximate representation of the original.

To find the external work done during any change,

measure the heat added under the curve with a planimeter and note the position of the beginning and end of the curve with reference to the constant intrinsic-energy curves. If this shows an increase, subtract, if a decrease, add the difference to the heat area and the final result will be the work area.

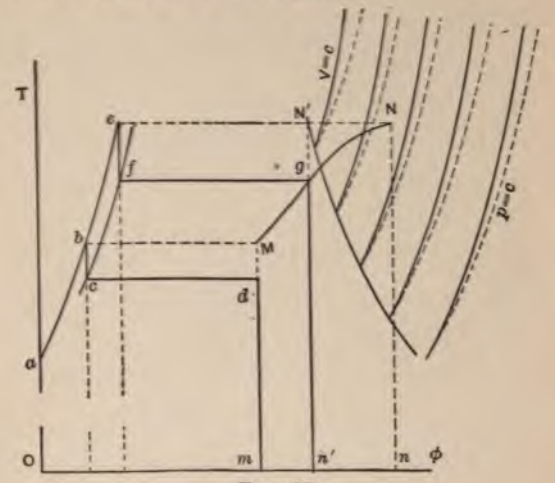


FIG. 15.

For a perfect gas the isothermal and the isodynamic lines coincide, and for superheated vapors the deviation is not excessive, so that for short range of temperature only a small error will be introduced in using the isothermals in place of the isodynamics. Hence to find the intrinsic energy of any point N, Fig. 15, in the superheated region, draw the isothermal NN' and assume *the intrinsic energy* of N to be the same as that of N', *i.e., area Oaefgn'O.*

Then for any change, as MN, extending from saturated to superheated steam, the heat added will be represented by $mMNn$, the increase of intrinsic energy by $befgn'mdcb$, and the external work by the algebraic sum of these two areas.

CHAPTER V.

THE TEMPERATURE-ENTROPY DIAGRAM FOR THE FLOW OF FLUIDS.

In view of the rapidly increasing use of the steam-turbine it has become necessary to understand thoroughly what happens when steam flows through a nozzle. As preparatory to that we will discuss first the frictionless, adiabatic flow of steam through a flaring nozzle. Such a process is reversible and hence the adiabatic line is also isentropic.

The general formula for frictionless adiabatic flow is

$$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=Ap_1v_1-Ap_2v_2+AE_1-AE_2.$$

We have to distinguish the three cases: (1) expansion from saturated to saturated, (2) expansion from superheated to saturated, and (3) from superheated to superheated. Substituting the proper values of v and E gives for the three cases:

1) $$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=Ap_1(x_1u_1+\sigma)-Ap_2(x_2u_2+\sigma)+q_1+x_1\rho_1 -(q_2+x_2\rho_2),$$

$$= q_1 + x_1\rho_1 + Ap_1x_1u_1 - (q_2 + x_2\rho_2 + Ap_2x_2u_2 + A\sigma(p_1 - p_2),$$
$$= q_1 + x_1r_1 - (q_2 + x_2r_2) + A\sigma(p_1 - p_2),$$
$$= H_1 - H_2 + A\sigma(p_1 - p_2);$$

2) $$A\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = Ap_1(u_1 + \sigma_1) + Ap_1(v_1 - s_1) - Ap_2(x_2u_2 + \sigma) + q_1 + \rho_1 + \frac{Ap(v_1 - s_1)}{k-1} - q_2 - x_2\rho_2,$$
$$= q_1 + r_1 + c_p(t_s - t_1) - (q_2 + x_2r_2) + A\sigma(p_1 - p_2),$$
$$= H_1 - H_2 + A\sigma(p_1 - p_2);$$

3) $$A\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = Ap_1(u_1 + \sigma) + Ap_1(v_1 - s_1) - [Ap_2(u_2 + \sigma) + Ap_2(v_2 - s_2)] + q_1 + \rho_1 + \frac{Ap_1(v_1 - s_1)}{k-1} - \left[q_2 + \rho_2 + \frac{Ap_2(v_2 - s_2)}{k-1}\right],$$
$$= q_1 + r_1 + c_p(t_{s_1} - t_1) - [q_2 + r_2 + c_p(t_{s_2} - t_2)] + A\sigma(p_1 - p_2),$$
$$= H_1 - H_2 + A\sigma(p_1 - p_2).$$

In each case the heat equivalent of the increase in the kinetic energy of the jet is equal to the decrease in the total heat of the steam plus the heat equivalent of the amount of work which would be required to

force a pound of water back against the difference of pressure.

Let ab, AB, and A_1B_1 in Fig. 16 represent three such expansions respectively.

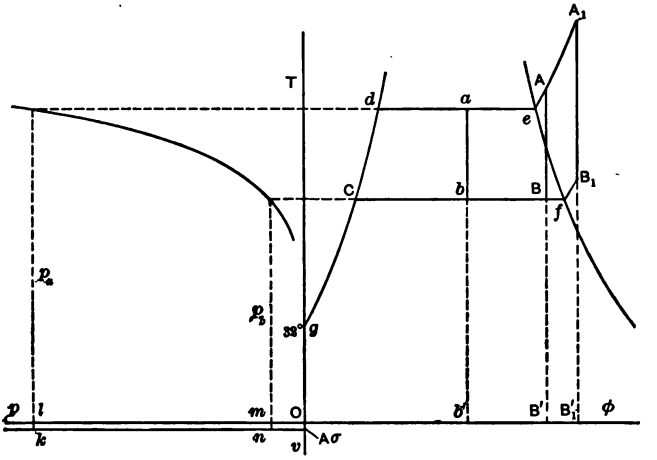


FIG. 16.

On the $T\phi$-diagram the increase in the kinetic energy of the jet between the pressures p_a and p_b is thus:

1) $$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=\text{area } Ogdab'O$$
$$-\text{area } Ogcbb'O+\text{area } klmn.$$

2) $$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=\text{area } OgdeAB'O$$
$$-\text{area } OgcBB'O+\text{area } klmn.$$

3) $$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=\text{area } OgdeA_1B_1'O$$
$$-\text{area } OgcfB_1B_1'O+\text{area klmn.}$$

That is, $A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$ is represented by the area enclosed between the water-line, the two constant-pressure curves, and the isentropic expansion curve plus the rectangle $klmn = A\sigma(p_a - p_b)$. Unless the steam is initially very wet this last term is inappreciable, so in general the heat equivalent of the increase in kinetic energy is equal to the difference in the total heats.

Suppose now we wish to design a nozzle to permit the flow of G pounds of steam per second. At any cross-section of area F the necessary and sufficient condition for continuity of flow is that

$$G = F\frac{V}{v} \quad \text{or} \quad \frac{F}{G} = \frac{v}{V}.$$

For any pressure p_x there can be only one definite value for velocity and specific volume, viz., V_x and v_x, and hence a unique value of the area F_x. The value of v_x may be read directly from the constant-volume curves; the value of V_x must either be computed from the above formulæ or else the area $cdAB$ measured by planimeter and V_x computed from that.

In Fig. 17 let $a'b'$ be the pv-projection of the frictionless adiabatic flow ab. Let MN represent the relative variations of specific volume and velocity during this expansion. Draw the line yy parallel to Ov and make Oy equal to G. Then at any point of the expansion as d, the volume $v_d = md'$ and the velocity $V_d = nd''$.

Draw Od'' and prolong if necessary until it intersects yy in k. From the similar triangles Oyk and $Od''n$ it follows that

$$yk : yO = On : nd''$$

or

$$yk = yO \cdot \frac{On}{nd''} = G \cdot \frac{v_d}{V_d} = F .$$

Conversely to find the pressure which would be obtained at any cross-section, lay off $yk = F$, draw kO,

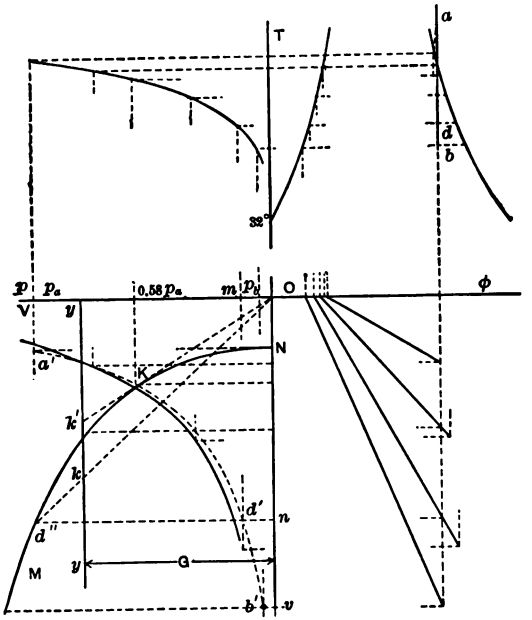


FIG. 17.

and prolong until it intersects MN in d'', from d'' drop *the perpendicular* $d''n$ and project d' back to d.

The smallest cross-section, or the throat of the nozzle, will be reached when Ok is tangent to MN, viz., Ok', giving the value $F_{throat} = yk'$.

As Ok cuts the vV-curve between K and N the value of F increases rapidly until at N it becomes infinite, which agrees with the initial assumption that $V_a = 0$. It is to be noticed that the throat is reached when the pressure has dropped to about 0.58 p_a. From this point on the nozzle flares indefinitely as long as the back pressure is dropped.

Expansion Through a Porous Plug.—So far in speaking of adiabatic lines reference has been made only to reversible processes; that is, the expansion was frictionless and work was done at the expense of the internal energy either upon a piston or in imparting kinetic energy to the molecules of the expanding fluid. Suppose now that the adiabatic expansion occurs through a porous plug (as in Kelvin and Joule's experiments with gases) so arranged that as soon as velocity dV is developed, it is at once dissipated through friction into heat dQ, which is returned to the body at the lower pressure $p - dp$. The first operation is reversible and hence isentropic, the latter is equivalent to the addition of the heat dQ from some external source and hence the entropy increases by the amount $\frac{dQ}{T}$. If this process is continued, instead of the adiabatic *coinciding* with the isentrope ab (as in reversible pro-

cesses) it takes the form ab', showing increasing entropy accompanied by falling temperature.

An entirely new interpretation must be given to the $T\phi$-diagram for irreversible processes such as this. The area under the curve ab' (Fig. 18) no longer represents heat added from external sources (nor from any other source). In this particular process no heat

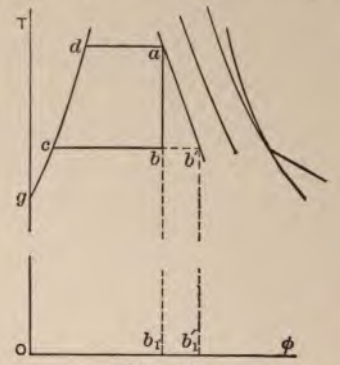


FIG. 18.

whatever was added. Suppose the change from a to b' is made by means of a frictionless, adiabatic expansion in a nozzle form a to b, producing the kinetic energy $abcd$, and that this energy is then reconverted into heat form at the lower pressure, represented by the change bb'. The increase in entropy is equal to $\frac{abcd}{bb_1}=\frac{bb'b_1'b_1}{bb_1}=bb'$. The area $bb'b_1'b_1$ represents the work lost in friction, and in this particular case is *equal to* all the work which might have been obtained *by a frictionless* adiabatic or isentropic change.

Making the final and initial velocities the same by proper proportioning of the plug, so that there is no increase in the kinetic energy of the fluid,

$$Ogcdab_1 + A\sigma(p_a - p_b) = Ogcb'b_1',$$

or practically

$$q_a + x_a r_a = q_b + x_b r_b = H;$$

i.e., ab' is approximately a curve of constant total heat. A series of such curves may be drawn as indicated, and the value of H for each curve will be that of λ at its point of intersection with the dry-steam curve. Extending the curves into the superheated region there must exist the following relation between points representing moist, dry, and superheated steam:

$$q_1 + x_1 r_1 = q_2 + r_2 = q_3 + r_3 + c_p(t_s - t_3) = H.$$

Peabody Calorimeter.—This principle is utilized in the Peabody throttling calorimeter. It is assumed that there is no increase in kinetic energy, hence

$$A\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = O = q_1 + x_1 r_1 - [q_2 + r_2 + c_p(t_s - t_2)] + A\sigma(p_1 - p_2),$$

or neglecting $A\sigma(p_1 - p_2)$,

$$q_1 + x_1 r_1 = \lambda_2 + c_p(t_s - t_2),$$

whence

$$x_1 = \frac{\lambda_2 + c_p(t_s - t_2) - q_1}{r_1}.$$

It should be noted that the transformation inside the calorimeter is not represented by the process ab', but more nearly by $ab+bb'$. Simply the initial and final conditions are represented by two points upon the same constant heat curve.

An inspection of the constant heat curves shows that starting with a given per cent. of priming in the steam-pipe, a reduction of the calorimeter pressure will cause more and more of the moisture to be evaporated until eventually the steam, with dropping pressure, becomes dry and then superheated. If at the lowest available back pressure the steam is not already superheated this calorimeter fails to impart the desired information. The higher the initial pressure the greater the percentage of moisture which the calorimeter can measure.

Flow Through a Nozzle.—In the case of the flow through an actual nozzle the operation is not reversible. Heat is lost by radiation; heat is conducted through the metal of the nozzle from the higher to the lower temperatures; and friction occurs in varying amounts in different parts of the nozzle. The first loss, the rejection of heat as heat, *decreases* the entropy of the fluid, while the other two losses both increase its entropy. It might happen that these opposing forces just balanced and then the expansion would be isentropic but not reversible. In general, however, the radiation loss may be made small, so that the operation *is nearly adiabatic*, but with increasing entropy

due to conduction along the nozzle and to friction losses.

Thus starting from *a*, Fig. 19, the actual expansion curve will lie between the isentrope *ab* and the con-

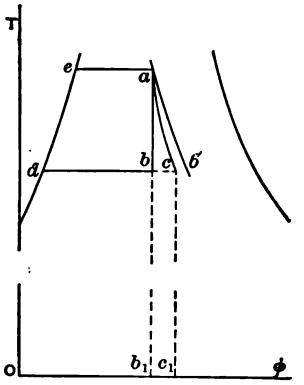


FIG. 19.

stant-heat curve *ab'* in some such position as *ac*. The heat theoretically available is represented by *abde*. By friction, etc., the portion bcc_1b_1 has been returned to the substance at a lower temperature, hence the kinetic energy of the jet at the exit *c* is equal to

$$\text{area } abde - \text{area } bcc_1b_1.$$

This loss would make itself noticeable in two ways. Decreased kinetic energy means decreased velocity, and increased entropy means increased volume. That is, if a nozzle were constructed from the dimensions necessary to give frictionless adiabatic flow and drilled *at different* points so as to measure the pressure, the

observed pressure would be found to be greater at any given cross-section than the pressure for frictionless flow. Stodola's experiments show this, and also that the loss is at first slight, being practically negligible down to the throat, but increasing from there onward more and more rapidly as the velocity increases. That is, the curve *ac* would at first closely approximate *ab*, but lower down branch off more and more toward the right.

As soon as the curve *ac* has been accurately located it can be projected into the *pv*-plane and the corresponding areas for different cross-sections of the nozzle determined in the manner already indicated for the ideal case of frictionless flow.

CHAPTER VI.

THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO HOT-AIR ENGINES.

THE Carnot cycle in the $T\phi$-plane is always a rectangle, but in the pv-plane its shape depends upon the nature of the working substance. For perfect gases the isothermals and frictionless adiabatics have nearly the same slope, so that to obtain an appreciable work area either the diameter of the cylinder or the length of the stroke must be made excessively large. That is, the excessive size and weight of the engine combined with large radiation and friction losses make the use of the Carnot cycle unfeasible in the case of hot air. Hence recourse has been had to certain of the isodiabatic cycles in the attempt to improve the work diagram.

The ideal cycle for the Stirling hot-air engine consists of the following events:

(1) Heating at constant volume by passage of air through regenerator.

(2) Expansion at constant temperature in contact with the hot surface of the furnace.

(3) Cooling at constant volume by return through the regenerator.

(4) Compression at constant temperature in contact with the cooling pipes.

The diagrams for such a cycle are shown in Fig. 20.

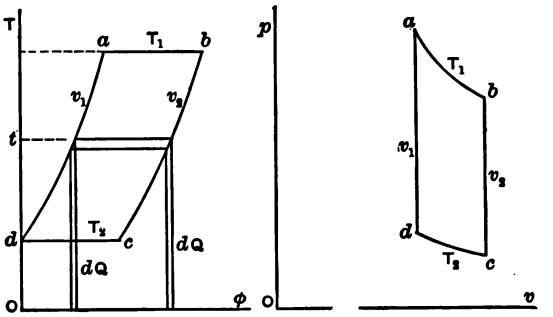


FIG. 20.

The criterion for such a cycle is that the heat rejected at any temperature T along bc shall equal that received at the same temperature along da. Hence

$$c_v dt + (c_p - c_v) T \frac{dv_1}{v_1} = dQ = c_v dt + (c_p - c_v) T \frac{dv_2}{v_2}.$$

As these equations both refer to the same isothermal it follows that

$$\frac{dv_1}{v_1} = \frac{dv_2}{v_2}, \quad \text{or} \quad \log v_1 + \log c_1 = \log v_2 + \log c_2,$$

whence

$$c_1 v_1 = c_2 v_2 \quad \text{or} \quad v_1 = c v_2.$$

That is, the lines *ad* and *bc* are "isodiabatic," as they satisfy the condition that the ratio of the volumes at the points of intersection with any isothermal is a constant.

The ideal cycle of the Ericsson engine is similar to that of the Stirling except that the heating and cooling occur at constant pressure instead of at constant volume.

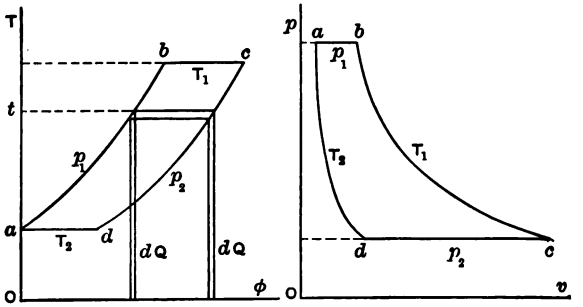


FIG. 21.

The ideal diagrams for such a cycle are shown in Fig. 21. In this case

$$c_p dt - (c_p - c_v) T \frac{dp_1}{p_1} = dQ = c_p dt - (c_p - c_v) T \frac{dp_2}{p_2},$$

whence $$p_1 = c p_2.$$

Hence these curves are "isodiabatic," since the ratio of the pressures is a constant.

Both the Stirling and the Ericsson cycle give well-*shaped* indicator-cards and are thus better than the *Carnot* cycle mechanically.

CHAPTER VII.

THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO THE GAS-ENGINE.

No attempt will be made here to trace the heat losses due to radiation and the cooling water in the jackets, but the cylinder and piston will be considered impermeable to heat in all cases. Thus by a comparison of the ideal cards for different cycles the gain due to initial compression and the loss from incomplete expansion may be more clearly defined.

The Lenoir cycle was introduced in 1860. Its thermodynamic principles were retained in the different free-piston engines. These were uneconomical and noisy and have disappeared. The only remaining example is the Bischoff, a simple small vertical engine.

The Lenoir cycle consists of the following events:

(1) During the first part of the forward stroke a fresh explosive mixture is drawn in by the piston (*aA* in Fig. 22).

(2) A little before half-stroke is reached the supply-*valve closes and the* explosion occurs. In reality the

combustion requires an appreciable time for its completion, and thus the heating takes place while the piston is moving forward, i.e., at increasing volume, but for the ideal case the explosion will be considered instantaneous, and hence the heating will be at constant volume; along the line *AB*.

(3) The rest of the stroke represents adiabatic expan-

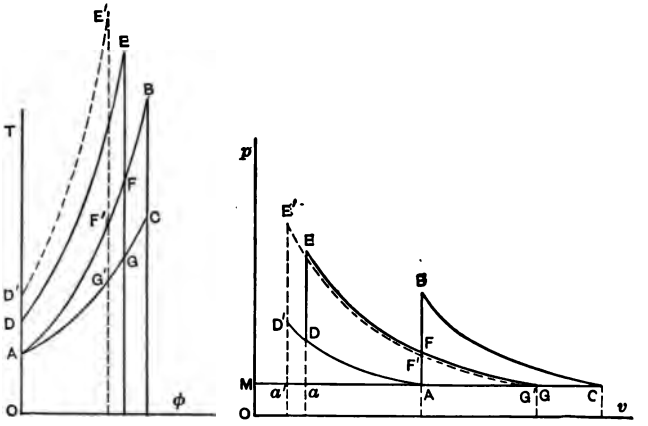


FIG. 22.

sion of the heated gas down to initial pressure; along *BC*.

(4) The return stroke, during which the products of combustion are exhausted. Thermodynamically this is equivalent to cooling at constant pressure; along *CA*.

About the time Otto and Langen were experimenting with the free-piston engine, Beau de Rochas described a cycle which would make possible the economical

running of a gas-engine. This was embodied by Dr. Otto in his silent engine in 1876 and has thus become associated, although wrongly, with his name.

The "Otto" cycle consists of the following events:

(1) The drawing into the cylinder at atmospheric pressure of a new explosive mixture throughout one complete stroke (*aA* in Fig. 22). The volume of the charge is *MA* and consists of the burnt products in the clearance space *Ma* from the last charge plus the fresh charge.

(2) The adiabatic compression of this charge on the return stroke of the piston *AD*. This compression of the gas into the clearance space is done at the expense of the energy in the fly-wheel.

(3) The ignition and explosion of the charge while the piston is at rest at the dead-centre, thus increasing the pressure and temperature at constant volume; along *DE*. Assuming that the same quantity of mixture is used by both the Lenoir and Otto engine, the heat generated by the explosion will be the same in both cases, i.e., the areas under the curves *AB* and *DE* are equal in the $T\phi$-diagram.

(4) The expansion of the heated gases throughout the entire stroke, assumed adiabatic; along *EF*.

(5) The drop in pressure due to the opening of the exhaust-valve while the piston is at the end of the stroke. This is equivalent to cooling at constant volume; along *FA*.

(6) The exhaust of the burnt gases during the return stroke *Aa*. Changes of location are not recorded in the $T\phi$-diagram.

In the Atkinson engine, now no longer made, the cycle was the same as the Otto up to the point *F*, and then, instead of releasing the hot gas, the expansion stroke was lengthened by means of an ingenious mechanism permitting the adiabatic expansion down to back pressure, as represented by *FG*. Then the exhaust stroke was from *G* to *A*, which thermodynamically is equivalent to cooling at constant pressure.

Comparing the Atkinson and Otto cycles it is at once evident that there is a loss of work and of heat equal to *AFG* in the *pv*- and $T\phi$-planes, respectively, due to incomplete expansion.

A comparison of the Atkinson and Lenoir cycles shows that as the heat received in both is the same while that rejected by the Lenoir engine is the greater (compare areas under *CA* and *GA*), the efficiency of the Atkinson is the greater.

Theoretically, then, the Atkinson engine has the most perfect cycle of the three, but nevertheless it has been entirely superseded by the Otto engine. The reason for this becomes at once apparent from the diagram. Even if the area *AFG*, rejected by the Otto engine, due to incomplete expansion, were just equal to the *area under GC* of the Lenoir exhaust stroke, the Otto

would still be preferable to the latter because the same amount of power could be developed with a smaller engine. Suppose, now, the clearance space in the above Otto engine were decreased, so that the adiabatic compression would heat the gas to a higher initial temperature, as AD'. The explosion would now occur along the constant-volume curve $D'E'$, where the area under $D'E'$ is equal to that under DE, as the same heat is generated in both cases. The adiabatic expansion would now be down $E'F'$ and the exhaust would be along $F'A$. Hence the same Otto engine with increased initial compression due to decreased clearance would give increased efficiency, as the heat rejected under $F'A$ is less than that rejected under FA. This engine would now be better than the Lenoir, both mechanically and thermodynamically. Furthermore the loss due to incomplete expansion becomes less because the heat thus rejected is reduced from AFG to $AF'G'$. That is, the higher the initial compression the less the theoretical superiority of the Atkinson over the Otto engine. And in the actual engine the increased complexity, size, friction loss, and danger of the Atkinson more than counterbalanced the theoretical superiority. Hence the Otto engine is practically the most efficient of the three.

In the Otto cycle the temperature at the end of compression is not very high relatively, so that during the first part of the combustion the working fluid is

much colder than the source of heat and does not attain this high temperature until the end of the combustion is reached. The heat is thus received at constant volume and increasing temperature instead of at constant temperature and increasing volume. It was shown in the first chapter that any such deviation from a Carnot cycle means a drop in efficiency. This led Diesel to invent a cycle during which most of the heat should be received at the highest available temperature. His method is to compress the air initially up to about five hundred pounds pressure to the square inch, so that its temperature is above the ignition-point of the combustible to be used. The injection of a small quantity of fuel causes the temperature to increase still further at constant volume up to that of the combustion; then as the piston moves forward the temperature of the gas is maintained nearly constant by the injection and combustion of further fuel. This lasts for about one-tenth of the stroke. The indicator-cards taken from such a motor show that the desired regulation is not perfect, the temperature sometimes rising, sometimes falling. This, however, only affects the magnitude of the gain from such a cycle, as all of the heat generated on the forward stroke is transmitted to the working fluid at an efficiency corresponding to that of the upper part of the Otto cycle. The cards show also that the expansion may or may *not be carried* down to the back pressure.

Fig. 23 shows the ideal diagrams for the Otto, Atkinson, and Diesel cycles for the same quantity of heat; that is, the areas under bc and $b'gc'$ are the same in the $T\phi$-plane. The change from b to b' shows the increased compression in the Diesel motor, b' being at or above the temperature of ignition. The heat received

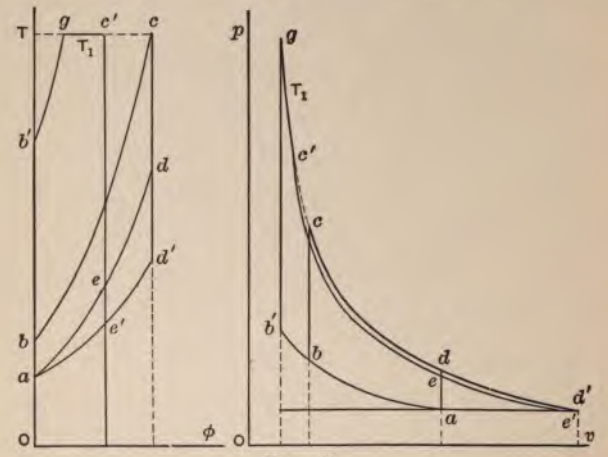


FIG. 23.

along $b'g$ is not received under the most efficient conditions, but still with an efficiency equal to that of the best part of the Otto cycle, while that received along the "isothermal combustion line" gc' is obtained under conditions of maximum efficiency. The effect, as is clearly shown by the diagram, is to increase the amount of heat changed into work and to diminish the heat *rejected*. On the return stroke the conditions of the Carnot cycle are more closely approximated in

the Atkinson and Diesel cycles where the heat is rejected at constant pressure than in the Otto cycle where it is rejected at constant volume, as lines of constant pressure deviate from isothermals less than do lines of constant volume.

CHAPTER VIII.

THE TEMPERATURE-ENTROPY DIAGRAM APPLIED TO THE NON-CONDUCTING STEAM-ENGINE.

THE Carnot cycle for steam gives a very good *pv*-diagram, and hence there are not the same mechanical objections to its adoption as in hot-air engines. But, due to the physical change in the working fluid, a differ-

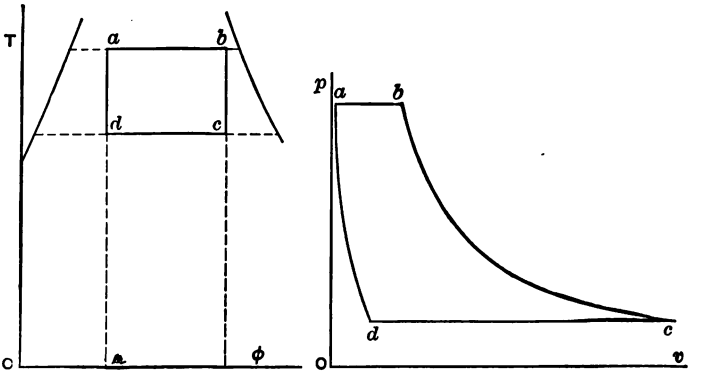


FIG. 24.

ent cycle has proved to be more feasible. In the Carnot engine the steam at condition *d*, Fig. 24, would *be compressed adiabatically* to *a* with the change in

condition from x_d to x_a. The isothermal expansion *ab* occurring by the application of heat to the cylinder produces the further change in condition to x_b. The cycle is finished by the adiabatic expansion *bc* and the isothermal compression *cd* with the cylinder in contact with the refrigerator.

The card of the ideal engine differs materially from this. (1) The line *ab*, Fig. 25, represents the admission

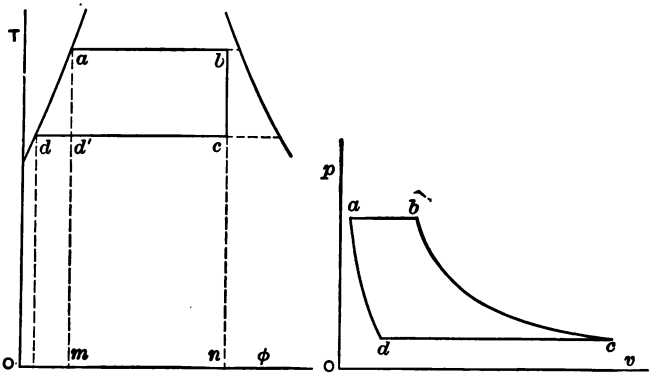


FIG. 25.

of steam of condition x_b into the cylinder up to the point of cut-off. This is forced in by the evaporization of an equivalent amount in the boiler, so that the $T\phi$-curve is the same as in the Carnot engine. (2) The adiabatic line *bc* represents the expansion of the steam admitted along *ab* plus the amount already in the clearance space at the moment of admission *a*. *(3) During* exhaust the piston simply forces out into the

condenser all of the steam taken during admission, but the quality of the *remaining* portion is the same at compression as at the beginning of release. (4) The part confined in the clearance space is then compressed along *da* in the *pv*-plane up to the initial pressure, i.e., back along the curve *cb* in the $T\phi$-plane. Hence, in a non-conducting engine, the amount of steam confined in the clearance space is immaterial, as its expansion and compression occur along the same adiabatic and do not affect the heat consumption.

That part of the steam exhausted during release, however, passes into the condenser and there condenses and gives up its heat to the cooling water. This is represented by *cd*.

From the condenser the water is forced into the boiler by means of a feed-pump, and is there warmed from *d* to *a* and vaporizes from *a* to *b*. The *pv*-diagram gives a history of the work performed per stroke and is confined entirely to the events in the cylinder. The $T\phi$-diagram, however, represents the heat cycle, and consists of events occurring in three different places. *da* and *ab* represent the heating of the feed-water and its evaporation at working pressure in the boiler, *bc* represents the adiabatic expansion in the cylinder of the engine, and *cd* the discharge of heat to the condenser.

If it were desired to make this cycle into a Carnot, *the condensation* would have to stop at d' and the

feed-pump arranged to compress the mixture adiabatically to a.

Suppose each engine to use one pound of steam of condition x_b, Fig. 25, per stroke, then the efficiency will be

a) Carnot: $$\eta = \frac{(x_b - x_a)r_1 - (x_c - x_d')r_2}{(x_b - x_a)r_1};$$

$$= \frac{\text{area } abnm - \text{area } d'cnm}{\text{area } abnm} = \frac{T_1 - T_2}{T_1};$$

b) Non-conducting or Rankine engine:

$$\eta = \frac{[q_1 - q_2 + (x_b - x_a)r_1] - (x_c - x_d)r_2}{q_1 - q_2 + (x_b - x_a)r_1}$$

$$= \frac{q_1 + x_b r_1 - q_2 - x_c r_2}{q_1 + x_b r_1 - q_2} = 1 - \frac{x_c r_2}{q_1 + x_b r_1 - q_2}.$$

An inspection of the diagram shows at once that

$$\eta_{\text{Carnot}} > \eta_{\text{Rankine}}.$$

It is evident also that for any given boiler pressure, the less the amount of moisture in the steam the smaller the difference between the Carnot and the Rankine cycles.

Increased Efficiency by Use of High-pressure Steam.—If the same quantity of heat be supplied per pound of steam under constantly increasing pressure the state point, b, Fig. 26, will assume the successive posi-

tions b', b'', etc., along the constant heat curve bb'', and at the same time the state point c, representing the condition at the end of the adiabatic expansion to the back pressure, moves towards the left into the successive positions c', c'', etc. Now the areas under cd, $c'd$, $c''d$, etc., represent the quantity of heat dis-

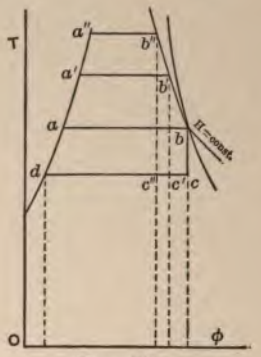


FIG. 26.

charged to the condenser under the different conditions. Therefore the greater the pressure or the higher the temperature at which a given quantity of heat is supplied to the engine, the smaller the fractional part rejected to the condenser, that is, the larger the portion turned into work and the greater the efficiency.

The Tp-curve shows that at high pressures the pressure increases much more rapidly than the temperature, and hence the necessary strength, weight, and cost of the engine will increase more rapidly than *the gain in efficiency*.

Gain in Efficiency from Decreasing the Back Pressure.—If the initial pressure be kept constant (Fig. 27) and the back pressure be diminished by increasing the vacuum, the heat taken up in the boiler by each pound of steam will be increased from *dabnm* to, say, *d'abnm'*, and the heat discharged to the condenser will

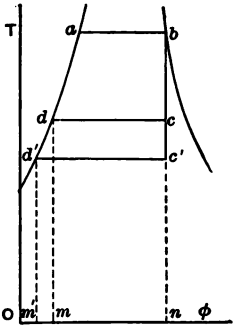


FIG. 27.

diminish from *dcnm* to *d'c'nm'*; that is, the efficiency increases.

Again referring to the *pT*-curve, it is clear that at low pressure the temperature decreases much more rapidly than the pressure, so that a small decrease in pressure means a considerable increase in efficiency. This is at once evident from an inspection of the efficiency for a Carnot cycle, $\eta=\frac{T_1-T_2}{T_1}=1-\frac{T_2}{T_1}$, but the expression for the Rankine cycle,

$$\eta=1-\frac{x_c r_2}{q_1+x_b r_1-q_2},$$

does not show the influence of the upper or lower temperature very clearly. The efficiency may easily be expressed as a function of the upper and lower temperatures by assuming the value of the specific heat of water to be constant and equal to unity. Thus

$$q_1 = \int_{32}^{t_1} c\,dt = T_1 - T_{32}, \quad q_2 = \int_{32}^{t_2} c\,dt = T_2 - T_{32},$$

and

$$x_c r_2 = \left(\theta_1 - \theta_2 + \frac{x_b r_1}{T_1}\right) \cdot T_2 = \left(\log_e \frac{T_1}{T_{32}} - \log_e \frac{T_2}{T_{32}} + \frac{x_b r_1}{T_1}\right) T_2$$

$$= \left(\log_e \frac{T_1}{T_2} + \frac{x_b r_1}{T_1}\right) T_2,$$

or

$$\eta = 1 - \frac{\left(\log_e \frac{T_1}{T_2} + \frac{x_b r_1}{T_1}\right) T_2}{T_1 - T_2 + x_b r_1}.$$

The value of the last term decreases, hence the efficiency increases,

(1) as x_b approaches unity,

(2) as T_1 increases, and

(3) as T_2 decreases.

Gain in Efficiency from Using Superheated Steam.—To avoid the introduction of excessively high pressures superheated steam is being used more and more. According to the Carnot cycle the gain in efficiency is equally great whether superheated or saturated steam of the

same temperature is used, but the Rankine cycle shows that the *theoretical* gain to be expected from superheated steam is but slight.

The portion *be* of the Rankine cycle, Fig. 28, represents the addition of heat in the superheater, and *ec* the expansion from superheated to saturated steam

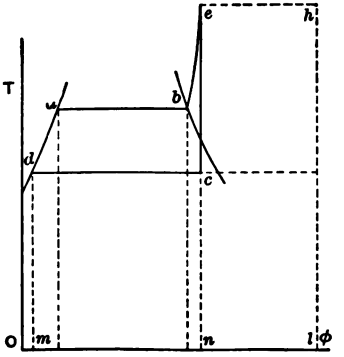


FIG. 28.

in the cylinder; the rest of the cycle is as previously described.

The heat $q_1 - q_2 + r_1 + c_p(t_s - t_1)$ is received along the line of varying temperatures *dabe*, while in the Carnot cycle an equal quantity of heat (area *ehln* = area *dabenm*) is all received at the upper temperature t_s. Hence the efficiency of the Rankine is now much less than that of the Carnot cycle working between the same temperature limits and the discrepancy increases as the *degree of superheating* increases.

The analytical formulæ for this case are:

$$\eta = \frac{q_1 - q_2 + r_1 + c_p(t_s - t_1) - x_c r_2}{q_1 - q_2 + r_1 + c_p(t_s - t_1)} = 1 - \frac{x_c r_2}{q_1 - q_2 + r_1 + c_p(t_s - t_1)}$$

$$= 1 - \frac{\left(\log_e \frac{T_1}{T_2} + \frac{r_1}{T_1} + c_p \log_e \frac{T_s}{T_1}\right) T_2}{T_1 - T_2 + r_1 + c_p(T_s - T_1)} \text{ (approximately).}$$

This shows an increase in efficiency with increasing T_s, but only of small amount.

It is evident, then, that the great gain obtained by using superheated steam must be looked for in the overcoming of certain defects inherent in an actual engine. The use of steam expansively entails a cooling of the working fluid and hence of the cylinder walls containing it. This effect is increased by release occurring before the expansion has reached the back pressure, and is only partially counteracted in part of the cylinder walls by the heating effect produced during compression. Thus the entering steam undergoes partial condensation before the cylinder walls have been brought up to its temperature; that is, each pound of steam, instead of occupying the volume which it had in the steam-pipe, now occupies a reduced volume proportional to the condensation. And hence, instead of obtaining the total area *abcd*, Fig. 29, only the fractional part *akld* can be utilized. Thus the *area lbcl has been* subtracted from the numerator of *the expression for* efficiency. This condensation may

range as high as from 20 per cent. to 50 per cent. of the total steam.

The addition of superheated steam may result in the superheat *bemn* being sufficient to supply the heat taken by the cylinder walls and thus preventing the condensation and making available the area *kbcl*. The economy is further increased as the steam at the

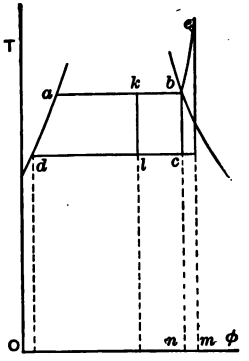


FIG. 29.

end of expansion has less moisture in it and thus abstracts less heat from the cylinder walls during release. That is, the conduction of heat through a vapor occurs but slowly, while water in contact with the metal will abstract large quantities of heat during evaporation. The leakage loss is also less with superheated steam.

Loss in Efficiency Due to Incomplete Expansion.— If steam be taken throughout the entire stroke the *indicator-card* is represented by abed (Fig. 30). The

drop in pressure *be* is equivalent to cooling at constant volume and may be represented on the $T\phi$-diagram by the curve of constant volume *be*. If the same quantity of steam be taken successively into larger cylinders, so that an increasing degree of expansion is obtained, this will be represented by *bc*, *bc'*, *bc''*, etc., in both diagrams. The areas *B*, *C*, *D* show the extra work performed per pound in the *pv*-plane, and the

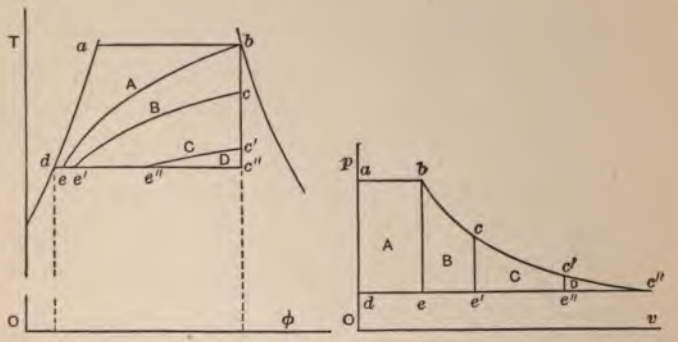


FIG. 30.

extra heat utilized in the $T\phi$-plane respectively, as the expansion progresses from initial to final pressure.

As in the gas-engine, so in the steam-engine it seldom pays to carry the expansion completely down to back pressure, because the slight gain from *c'* to *c''* is more than counterbalanced by the increased size, cost, and weight of the engine, friction, and radiation losses, etc.

For such incomplete expansion the expression for the *efficiency of the* Rankine cycle is found as follows:

$$Q_1 = q_1 - q_2 + x_1 r_1,$$

$$Q_2 = Q_{ce} + Q_{ed} = E_c - E_e + Q_{ed}$$

$$= (q_c + x_c \rho_c) - (q_2 + x_e \rho_2) + x_e r_2 + (A p_c x_c u_c - A p_c x_c u_c)$$

$$= q_c + x_c r_c - q_2 + A x_c u_c (p_c - p_2),$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{x_c r_c + q_c - q_2 - A x_c u_c (p_c - p_2)}{q_1 - q_2 + x_1 r_1}.$$

Loss in Efficiency from Use of Throttling Governor.—The throttling governor acts by wire drawing the steam to a lower pressure. Less steam is thus taken per stroke, as the volume is increased by both the reduced pressure and the increased value of x. A series of cards for dropping pressure is shown in Fig. 31. The

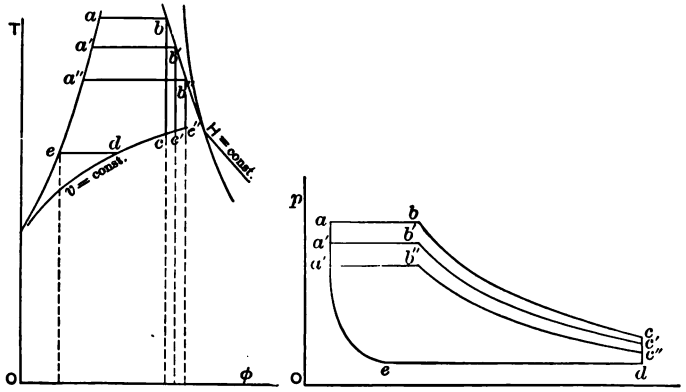


FIG. 31.

$T\phi$-diagram shows the decreased efficiency *per pound* of steam for the same cases. During wire drawing the *heat remains* the same, but the entropy increases, as

the process is irreversible. The heat rejected increases as the initial pressure drops, so that of the total heat brought in a smaller quantity is changed into work and the efficiency of the plant decreased.

CHAPTER IX.

THE MULTIPLE-FLUID OR WASTE-HEAT ENGINE.

In the discussion of the Rankine cycle it was shown how the efficiency of the steam-engine could be increased by raising the temperature of the source of heat or by decreasing that of the refrigerator. Due to the course of the pT-curve a practical upper limit is soon reached in the use of saturated steam due to the rapid increase of pressure at upper temperatures, so that recourse has to be taken to superheated steam. Again, in reducing the back pressure a slight drop in pressure means a large drop in the exhaust temperature, but a practical limit is soon reached beyond which it does not pay to carry a vacuum.

Theoretically, at least, the efficiency could be increased by using for the higher temperatures some fluid (X) having a smaller vapor pressure than saturated steam, and for lower temperatures some fluid (Y) having a greater vapor pressure than saturated steam at the same temperature. That is to say, the first fluid X could in a saturated condition be heated to the temperature now common for superheated steam, and then be

allowed to expand in a cylinder down to some lower temperature at which the pressure of saturated steam would not be excessive. The surface condenser for this X-fluid would be at the same time the boiler for the steam. After the steam had expanded through two or three cylinders, it in turn would exhaust into a second surface condenser, which would be the boiler for the next fluid, Y, in the series. Thus the working substance in each case is condensed at the temperature of its own exhaust and fed back to its own boiler at this temperature. The heat of vaporization which the first fluid rejects must warm up the second as it is fed into the boiler and then vaporize part of it, so that

$$\text{heat rejected} = \text{heat received},$$

or
$$xr_{\text{exhaust}} = (q + xr)_{\text{boiler}} - q_{\text{condenser}}.$$

For the sake of simplicity suppose at first that two such fluids X and Y as described could be found, and further that their liquid and saturated-vapor lines coincided in the $T\phi$-plane with those for water, but that the Tp-curves are entirely different.

Suppose, further, that p_1 and p_2 (Fig. 32) represent respectively the highest pressure which it is convenient to use, and the lowest pressure which can be obtained in a vacuum. The range of temperatures when saturated steam alone is used is limited between t_1 and t_2. If, however, the fluid X is first used the upper temperature can be raised to t_X for the same maximum pres-

sure p_1. Suppose in this problem that the substance Y does not solidify at 32° F., its liquid line would extend to the left of the arbitrary zero for the entropy of water, and so the expansion of this fluid down to p_2 would drop the lower temperature from t_2 down to t_Y.

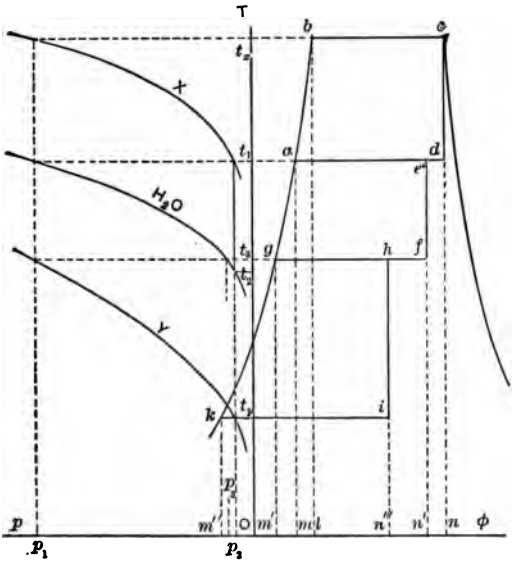


FIG. 32.

The liquid X is fed into its boiler at the temperature t_1, and is warmed along *ab*, receiving the heat $q_b - q_a$, equal to area *ablm*; it is then vaporized at the pressure p_1 and receives the heat r_X equal to *bcnl*. Its ideal cycle is now completed by adiabatic expansion *cd* down to some pressure p_X (in Fig. 32 p_X coincides *with* p_2), corresponding to temperature t_x, on the X-

curve, and condensation along *da*. The condensed fluid is at the proper temperature to be returned to its boiler.

The heat rejected along *da*, viz., $x_d r_d$, must warm up the water fed into the *X*-condenser at temperature t_3 up to t_1 and then vaporize part of it at the upper temperature. Assuming no heat lost,

$$\text{area } m'gaen' = \text{area } madn.$$

The steam describes the ideal cycle *gaef*, rejecting in turn the heat under *fg*, equal to $x_f r_f$ at some pressure p_3 corresponding to temperature t_3.

In the steam-condenser the fluid *Y* is first warmed up from the temperature t_Y, corresponding to p_2, to t_3 and then enough vaporized to make

$$\text{area } m''kghn'' = \text{area } m'gfn'.$$

From the diagram the following conclusions can be drawn:

Heat received from fuel equals *mabcn*.

Heat utilized by *X*-engine equals *abcd*.

Efficiency of *X*-engine, $\eta_X = \dfrac{abcd}{mabcn}$.

Heat rejected by *X*-engine = area *madn* = area *m'gaen'*
= heat absorbed by steam-engine.

Heat utilized by steam-engine equals *gaef*.

Efficiency of steam-engine $\eta_{st} = \frac{gaef}{m'gaen'}$.

Efficiency of X- and steam-engines combined equals

$$\eta_{(X+st)} = \frac{abcd + gaef}{mabcn}.$$

Heat received by Y-engine = heat rejected by steam-engine = $m''kghn'' = m'gfn'$.

Heat utilized by Y-engine = $kghi$.

Heat rejected by Y-engine = $m''kin''$.

Efficiency of Y-engine $= \frac{kghi}{m''kghn''}$.

Efficiency of all three engines together,

$$\eta_{(X+st+Y)} = \frac{abcd + gaef + kghi}{mabcn}.$$

The heat rejected has been reduced from

$$madn \text{ to } m''kihfednm''.$$

The great gain in efficiency shown in this assumed case is deceptive. The exhaust temperature has been taken far below freezing. This could not be done unless some cooling mixture could be employed in order to condense the exhaust fluid Y.

This would be expensive and probably represent as *great an* expenditure of work as the increased gain

recorded by the Y-piston, possibly more. In practice the lowest temperature t_Y will be governed by that of the cheapest available condensing substance, i.e., the temperature of the cooling water at the power-station.

The upper temperature t_X will be governed by the materials used in construction.

Since $x_{ex}r_{ex}=q_B-q_C+x_Br_B$, it follows that $x_B<x_{ex}$; that is, the value of x gradually grows smaller for succeeding fluids, so that initial condensation must be increasing in the successive cylinders. The gain which accrues from the use of superheated steam might for similar reasons be expected from the use of the superheated vapors of the X- and Y-fluids. It thus would undoubtedly pay to superheat each vapor as it leaves its respective boiler an amount sufficient to overcome the initial condensation. This might be effected by the use of a separately fired superheater suitably situated, or perhaps more economically still the hot flue gases from the first boiler furnace might be made to pass successively through all the superheaters and thus the total economy increased two ways at once.

Several different multiple-fluid engines have been proposed, usually of the binary type. When it comes to a discussion of any particular combination the actual $T\phi$-diagram must be drawn, and the liquid- *and saturated-vapor* lines will no longer be super-

imposed as was assumed in the ideal case just discussed. However, the principles already enunciated will make the application clear.

Any fluid having a low boiling-point, such as ammonia, chloroform, sulphur dioxide, ether, carbon disulphide, etc., is available for such work. All such volatile fluids possess latent heat of vaporization of small magnitude, and the smaller this is, the more volatile the substance and the greater its specific pressure at a given temperature. This leads to the practically important fact that to perform a given amount of work a greater quantity of the volatile substance must be supplied, the amount necessary increasing with the diminution of the latent heat of vaporization.

Du Trembley used a steam-ether engine. Ether superheats during adiabatic expansion and thus seems especially adapted to such work, as this would tend to prevent the excessive cooling of the cylinder during exhaust and thus do away with the losses incident to initial condensation. Fig. 10, although not drawn to accurate scale, gives an approximate idea of the relative values of the latent heat of vaporization. In round numbers the entropy as liquid and as vapor compares with water as follows:

$$\text{Water} \begin{cases} 32^\circ \text{ F.. } \theta = 0, & \theta + \frac{r}{T} = 2.12; \\ 248^\circ \text{ F.. } \theta = 0.365, & \theta + \frac{r}{T} = 1.69; \end{cases}$$

$$\text{Ether} \begin{cases} 32^\circ \text{ F.} .\theta = 0, & \theta + \frac{r}{T} = 0.34; \\ 248^\circ \text{ F.} .\theta = 0.205, & \theta + \frac{r}{T} = 0.385; \end{cases}$$

and as the latent heat is about one sixth that of water it follows that about six pounds of ether will be required to cool each pound of steam, so that a combined $T\phi$-diagram might be drawn for one pound of water and six of ether.

Perhaps the most accurate and elaborate series of experiments on any binary engine was made by Prof. Josse of Berlin. He used sulphur dioxide for the secondary fluid. The best results obtained were 11.2 pounds of steam per horse-power for the steam-engine alone and the equivalent of but 8.36 pounds per horse-power per hour using the combined engine. In the test the "waste-heat" engine added 34.2 per cent. to the power obtained from the primary engine.

The steam had a pressure of 171 pounds absolute and was superheated to 558° F. The back pressure on the low-pressure cylinder of the steam-engine was about 2.9 pounds absolute, corresponding to 140° F. The SO_2 cylinder received vapor under pressure of 143 pounds and exhausted at 48.2 pounds absolute, corresponding to a temperature of 67.8° F.

In round numbers the latent heat of SO_2 is about *one seventh that of* water, so that it would require the

vaporization of from 6 to 7 pounds of SO_2 to condense 1 pound of steam.

Fig. 33 shows the ideal cycle for a binary engine of this type working between the pressures and tem-

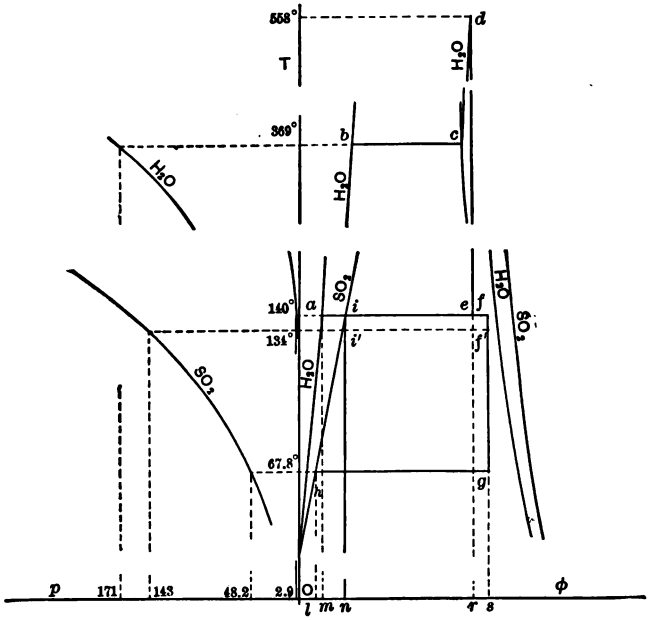


FIG. 33.

peratures realized by Prof. Josse in the test at Charlottenburg.

The steam-engine was triple expansion, and the ideal cards for such an engine are shown combined at *abcde*. The heat exhausted to the steam-condenser, or SO_2 boiler, *xr*, equals *aerm*. Of this quantity an

amount *hifg* is saved by the SO_2 cylinder theoretically. Actually there was a drop in temperature of about 6° F. between the low-pressure steam-cylinder and the SO_2 cylinder, so that the heat area *iff'i'* was either totally lost or partially reduced to lower efficiency by wire-drawing.

CHAPTER X.

LIQUEFACTION OF VAPORS AND GASES.

SUPERHEATED steam of the condition shown at a in Fig. 34 might be changed to saturated steam by one

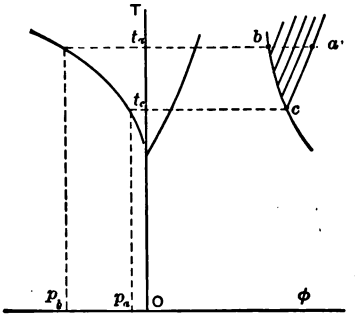


FIG. 34.

of two methods. First, it could be kept in a hot bath at constant temperature and the pressure increased from p_a to p_b, so that its state point would move from a to b. Secondly, it might be kept under constant pressure p_a (as a weighted piston) and its temperature allowed to drop by radiation from t_a to t_c, so that the *state point* travels from a to c. Liquefaction will

begin by either process as soon as the state point reaches the dry-steam line.

It is at once evident that the second method is the only one always applicable, for the isothermal change might take place above the critical temperature and then no increase of pressure, however great, could result in liquefaction.

The critical temperature of steam is beyond the upper limit of temperature used in engineering, so that this is not as clear here as in the case of some vapor which superheats at ordinary temperatures, as, for example, carbon dioxide.

Fig. 35 * shows the $T\phi$-, pv-, Tp-, and $v\phi$-diagrams for CO_2. The chief differences between this diagram and that for steam lie in the fact that the critical temperature is included, thus showing the intersection of the liquid- and saturated-vapor curves, and further, that the volume of the liquid is now appreciable with reference to that of the vapor and its variation with increasing pressure and temperature no longer negligible.

If a (Fig. 35) represent the state point of the superheated CO_2 vapor at the pressure and temperature under consideration, isothermal compression will fail to produce condensation, although cooling at constant pressure will produce liquefication as soon as c is reached.

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* This diagram is only approximately correct, being based *upon somewhat discrepant* data given by Amagat, Regnault, *and Zeuner.*

The ordinary gases, hydrogen, oxygen, nitrogen, etc., have their critical points as much below ordinary

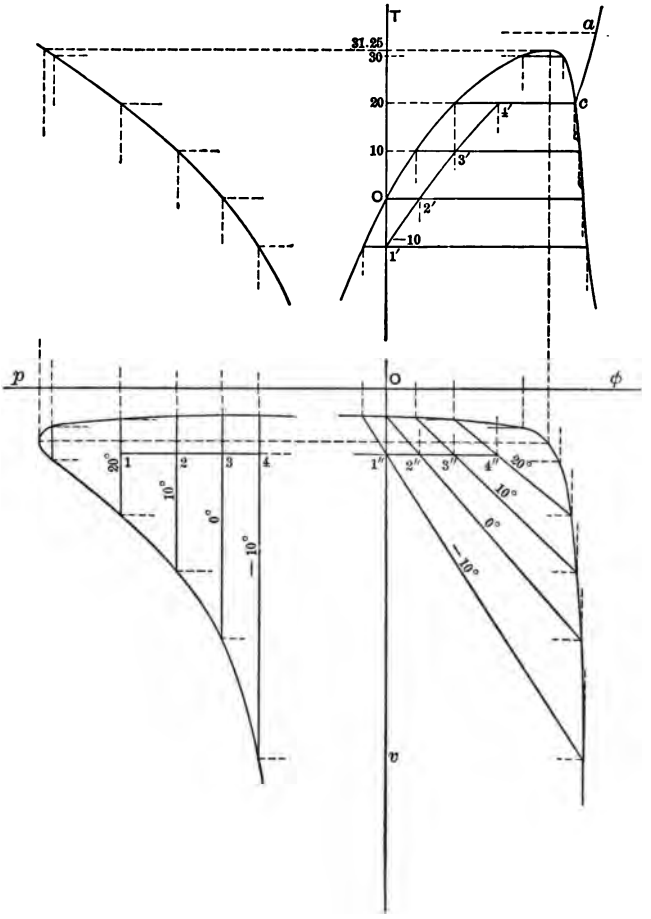

FIG. 35.

engineering temperatures as that of water is above *them.*

Superheated steam at ordinary temperatures could be liquefied isothermally because its temperature is less than that of the critical point; carbonic dioxide at ordinary atmospheric temperatures could also ordinarily be liquefied isothermally because the usual atmospheric temperature is less than 31.9° C. (89.4° F.); hydrogen, oxygen, etc., cannot be liquefied isothermally simply because the atmospheric temperature is far above their critical temperatures. The essential factor in liquefaction, then, is to reduce the temperature and then simply to compress the gas isothermally until liquefaction commences.

Suppose it is desired to liquefy some carbon dioxide some summer day when the temperature of the atmosphere is above its critical temperature. Let the compression be carried on slowly, so that the heat generated may be dissipated by radiation and the process be isothermal. Liquefaction will not occur. It will be necessary to cool the gas down to the critical temperature by some means, physical or chemical. Possibly a coil containing cold water will suffice in this case. Proceeding to other substances possessing lower and lower critical temperatures, cooling mixtures giving lower and lower temperature would be required. That is, by this method it would never be possible to liquefy any substance, however great the pressure applied, unless there already existed some source of cold as *low as its critical* temperature. When the "permanent"

gases are reached the sources of artificial cold fail, and unless the gas may be made to cool itself investigation must cease. Any new gas thus liquefied of course in turn becomes a new source of cold to aid in further investigation.

Let Fig. 36 represent the $T\phi$-diagram of some gas having a low critical temperature. Consider the

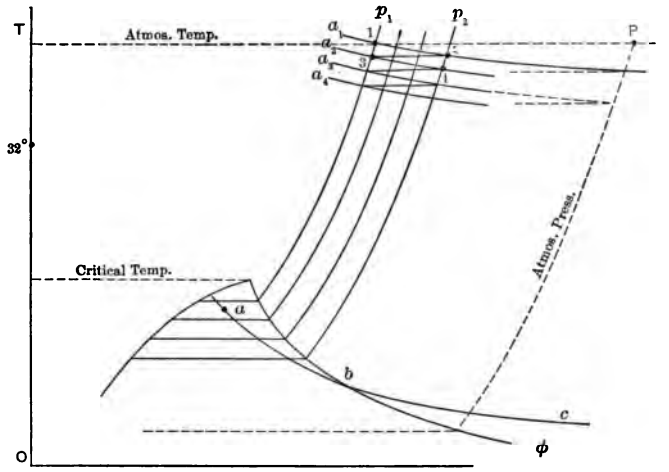

FIG. 36.

throttling-curve *abc*. By definition this represents an adiabatic change, during which no work is performed; i.e., the heat contained in the substance is a constant. For this curve the first law of thermodynamics gives

$$A\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)=O=A(E_1-E_2+p_1v_1-p_2v_2),$$

which becomes

$$E_1-E_2=p_2v_2-p_1v_1;$$

that is, the curve represents an irreversible isodynamic process in the case of a perfect gas and differs but slightly from it for ordinary gases.

Now, the internal energy of any substance is defined as the summation of the sensible heat and the disgregation work, or the kinetic energy of the molecules due to their own vibrations plus the potential energy due to their mutual positions. Representing these quantities by S and I respectively, it follows that $S_1+I_1-S_2-I_2=p_2v_2-p_1v_1$.

The work necessary to separate the molecules against their mutual attraction must increase with the distance between them although the rate of increase is inversely as the square of the distance between them.

As the volume increases the value of I increases and hence the value of S must decrease an amount equal to

$$I_2-I_1+p_2v_2-p_1v_1;$$

that is, the temperature of the substance decreases.* With increasing volume the rate of temperature drop decreases so that the curve *abc* approaches the ϕ-axis as an asymptote. Near the saturation curve, in the region of the "superheated" vapors, this drop is considerable, but far to the right of this curve and above the critical temperatures these "throttling" curves become almost parallel to the ϕ-axis. The "perfect

*For hydrogen at ordinary temperatures $p_2v_2-p_1v_1$ is negative and greater than I_2-I_1, so that the temperature increases.

gas" is simply the limiting condition in which the potential energy has attained its maximum value, or rather where any limited change in volume does not affect the total value of I appreciably.

In such a case

$$p_2v_2 = p_1v_1,$$

$$E_1 = E_2,$$

and

$$S_1 + I_\infty = S_2 + I_\infty;$$

that is, the disgregation change being negligible, the isodynamic curves become coincident with the isothermals.

In Fig. 36 let a_1, a_2, a_3, etc., represent a series of such throttling-curves. Let a pound of air be taken from its initial condition P (representing atmospheric pressure and temperature) and be compressed isothermally to 1. This may be effected by jacketing the cylinder walls of the compressor with cold water. If the air is then permitted to expand along the throttling-curve a_1, the temperature will drop, say, from 1 to 2. The air at reduced pressure is fed back to some intermediate stage of the compressor. In some forms of liquefiers the expansion is carried at once down to atmospheric pressure, thus securing a somewhat greater drop in temperature, and the air is then returned to the first stage of the compressor. If this cooled, expanded air be made to flow back outside the pipe containing more air from the compressor, this in turn will be

cooled by conduction to some lower temperature and so will escape from the throttling-valve at pressure p_1 at some lower temperature (3). This will now expand along the throttling-curve a_2 to a still lower temperature (4). This process is continued until the temperature of the issuing jet has fallen below the critical temperature, when liquefaction will ensue.

This will evidently occur first in the nozzle, as the temperature outside will need to be still further decreased before the air will remain liquid at the reduced pressure. The vaporization of the liquid first formed tends to still further decrease the temperature of the air-tubes, etc. It is probable that at first all of this liquid vaporizes as soon as ejected, but the vaporization of part soon cools down the rest and its surroundings, so that a small portion remains liquid at the lower pressure. The back pressure may thus in time be reduced to that at P and then the temperature will be found at which air vaporizes at atmospheric pressure.

The expansion of the air thus provides in itself the cooling process needed to reduce the temperature below the critical point so that sufficient increase of pressure may cause liquefaction.

CHAPTER XI.

APPLICATION OF THE TEMPERATURE-ENTROPY DIAGRAM TO AIR-COMPRESSORS AND REFRIGERATING-PLANTS.

In self-acting machines the working fluid is received at a high temperature, part of its heat changed into work and the remainder exhausted at a lower temperature. For the reverse operation, work has to be performed upon the machine, and the heat equivalent of this work added to the heat taken in at low temperature is rejected at a higher temperature. Such an operation is used to attain one of three results, viz.: (1) to store up power for immediate or future use, (2) to decrease the heat of the cold body, or (3) to increase the heat of the warm body.

The Compressor.—The cycle in the compressor is the same for all three cases, viz., the reverse of that in an engine, and hence a discussion of the work expended in the compressor will be given first. Referring to Fig. 37, the cycle is as follows: From *a* to *d* the gas (or vapor) in the clearance space expands until the *pressure at d* falls sufficiently below that in the sup-

ply pipe to permit the admission valve to open and admit a new supply from d to c. On the return stroke, the entire quantity is compressed along cb, until the pressure becomes sufficient to lift the release valve, when discharge occurs from b to a against the upper pressure. The indicator-card thus shows the entire cycle of the clearance gas, but only one portion, the

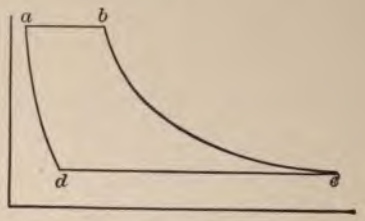

FIG. 37.

compression, of that of the charge. The expansion and compression of a gas in a cylinder is more nearly adiabatic than that of a saturated vapor, and hence, as the temperature of the gas at the end of admission is nearly that existing at the end of expansion, the expansion and compression of the gas in the clearance space may be considered to neutralize each other thermodynamically; although mechanically the greater the clearance the greater the size of the cylinder necessary to compress a given amount of gas. Therefore all discussion of the effects of clearance will be omitted in the following.

Due to the slow conduction of heat in gases, it is

probable that water-jacketing does not cause the compression to deviate much from adiabatic until high pressures are reached, with the consequent larger ratio of radiating surface to volume. In the above first and second cases the gas or vapor is cooled either unavoidably or intentionally to atmospheric temperature before being used as a source of power or as a refrigerant, so that any increase of temperature during compression represents wasted work.

In Fig. 38, let *ab* and *ac* represent isothermal and

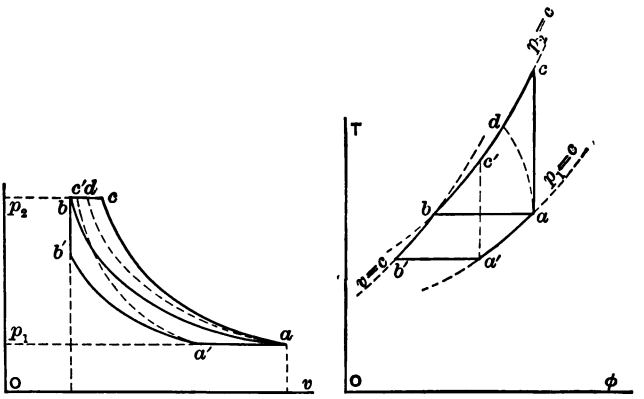

FIG. 38.

frictionless adiabatic compression respectively from some lower pressure p_1 to a higher pressure p_2. If the temperature of the cooling water is the same as that of the atmosphere, the minimum possible expenditure of work in compressing from *a* to *b* equals that under the *isothermal ab*, or $p_a v_a \log_e \frac{v_a}{v_b}$. If, however, the

compression is adiabatic it will be necessary for the delivered air to contract at constant pressure, losing by conduction and radiation the heat $C_p(t_c - t_b)$. That is, the work $\frac{p_a v_a}{K-1}\left[\left(\frac{p_c}{p_a}\right)^{\frac{K-1}{K}} - 1\right]$ is performed upon the gas during compression, as shown under *ac*, and the work $p_b(v_c - v_b)$ during contraction in the storage tubes or coil, as shown by *cb* in the *pv*-diagram. The wasted work is thus shown by the area *abc*, and has the value

$$\frac{p_a v_a}{K-1}\left[\left(\frac{p_c}{p_a}\right)^{\frac{K-1}{K}} - 1\right] + p_b(v_c - v_b) - p_a v_a \log_e \frac{p_b}{p_a}$$

$$= \frac{p_a v_a}{K-1}\left[\left(\frac{p_c}{p_a}\right)^{\frac{K-1}{K}} - 1\right] + p_a v_a\left[\left(\frac{p_c}{p_a}\right)^{\frac{K-1}{K}} - 1\right]$$

$$- p_a v_a \log_e \frac{p_b}{p_a}$$

$$= \frac{K}{K-1} \cdot RT_a\left[\frac{T_c}{T_a} - 1\right] - AT_a[\phi_a - \phi_b]$$

$$= AC_p[T_c - T_a] - AT_a[\phi_a - \phi_b].$$

In the $T\phi$-diagram the minimum amount of heat rejected is shown under *ab*, while that rejected during contraction at constant pressure after adiabatic compression is shown under *cb*, the heat wasted by the latter process being shown by *acb*. If, as is usually the case, the compression line lies somewhere between *these two* extremes, the wasted work and heat will be *represented by* some such area as adb.

If the cooling water is colder than the atmosphere, it is, theoretically at least, possible to reduce the necessary work by cooling the entering gas at constant pressure from a to a', compressing isothermally to b', and then permitting the gas to warm up at constant volume by taking heat from the atmosphere. The work performed and the heat rejected during compression are represented by the areas under $a'b'$ in the two diagrams, and the work saved by $aa'b'b$. If the compression from a' is along the adiabatic $a'c'$, the saving over that along ac is shown by the area $aa'c'c$.

It is not possible to cool the hot gas very much by jacketing, but the waste work may be reduced by dividing the compression into two or more stages and cooling the gas in intermediate coolers to the initial temperature.

Thus suppose the compression to follow the law $p_1v_1^n = p_2v_2^n$, and to be represented by the curve ac in Fig. 39. Instead of completing the compression in one cylinder, stop at some intermediate pressure p_x, at d, and cool under constant pressure to e. Continue the compression in a second cylinder along ef, and finally cool at constant pressure along fb. The wasted work or the heat ejected is no longer represented by the whole of abc, but by the two portions ade and efb; that is, the work or heat saved by compounding is represented by the area $cdef$. From the diagram it is at once evident that this area approaches zero as p_x

approaches either p_1 or p_2, and that there is some intermediate position which gives the maximum saving. The

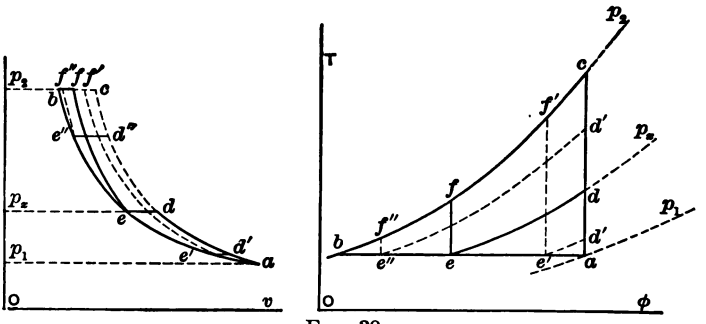

FIG. 39.

proper value of p_x is easily found from the expression for work

$$W = W_{ade} + W_{efb} = \frac{n}{n-1} \cdot p_a v_a \left[\left(\frac{p_x}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_x}\right)^{\frac{n-1}{n}} - 2 \right].$$

This expression has its minimum value when

$$\frac{d}{dp_x}\left[\left(\frac{p_x}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_x}\right)^{\frac{n-1}{n}} \right] = 0,$$

i.e., when $p_1 : p_x = p_x : p_2$, or $p_x = \sqrt{p_1 p_2}$.

Fig. 40 shows similar diagrams for a three-stage compressor with intercoolers. The saving thus introduced is shown by the irregular-shaped figure *defghc*. *This again varies* in magnitude with the values of

p_x and p_y, and in a manner similar to the above may be shown to have its maximum value when

$$p_1 : p_x = p_x : p_y = p_y : p_2,$$

or $$p_x = \sqrt[3]{p_1^2 p_2} \quad \text{and} \quad p_y = \sqrt[3]{p_1 p_2^2}.$$

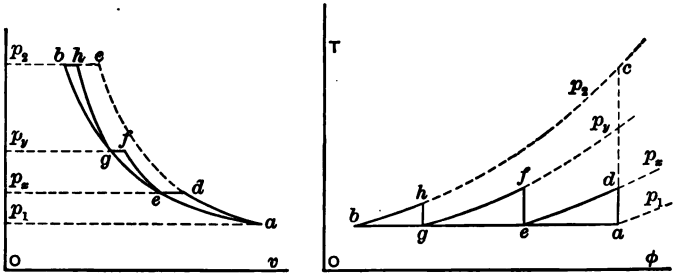

FIG. 40.

Compressed Air Used as a Source of Power.—Returning to the first general case where the air is used for power, it is necessary first of all to discuss the influence of the pipe line which conducts the air from the compressor to the machine to be operated by the compressed air, such as a rock-drill, a penumatic riveter, a compressed-air motor, etc. The temperature of the pipe may be considered as equal to that of the surrounding atmosphere and hence constant. The heat generated by friction is thus at once dissipated by conduction, and there thus results an isothermal drop of pressure and increase of volume.

Thus, if abcde, Fig. 41, represent the passage of the air

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through the compressor, *ef* shows the loss experienced by the air in flowing from the compressor to the engine. Suppose the air at *f* to expand adiabatically in the motor down to back pressure, the amount of work performed will be equal to the area under *fg* in the *pv*-diagram. The exhaust air is now warmed to the initial temperature along the constant pressure-curve *ga*, thus per-

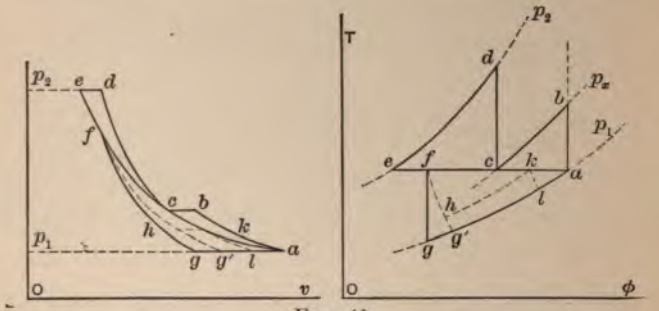

FIG. 41.

forming upon the atmosphere the work under *ga* in the *pv*-plane, and receives from the atmosphere the heat under *ga* in the $T\phi$-plane. The maximum amount of work could be obtained from such an engine if the expansion were along the isothermal *fa*. This can be partially attained by jacketing with water at atmospheric temperature, so that the actual expansion-curve lies somewhere between these two limiting cases, as at *fg'*. A further gain could be made by compounding the engine and heating the air up to atmospheric temperature in the intermediate receiver, as indicated by *fhkla*.

Such a means of obtaining power is not economical, but has many practical justifications, as, for example, in underground work, where exhaust-steam would be disadvantageous, or for long-distance transmission, where steam would be wasteful, due to condensation losses, etc.

Refrigerating plants may be subdivided under three heads: those using a compressor with (1) air or (2) some saturated vapor—usually ammonia—as the agent, and

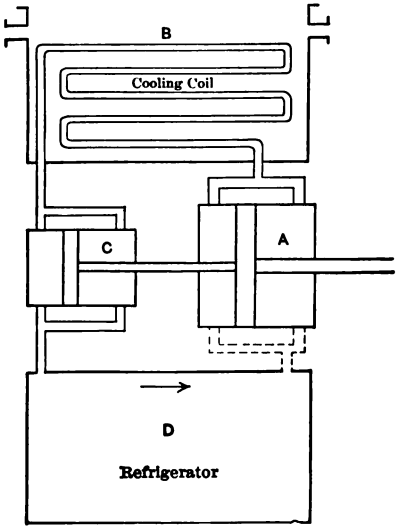

FIG. 42.

(3) absorption plants. The last will not be discussed here.

Air Refrigeration. — In air refrigerating plants the *work performed* during expansion is no longer the

main object, as in the case just discussed, but is simply a means of obtaining the desired end, viz., a sufficient drop in the temperature of the air. The essential parts of such a system are shown in Fig. 42. The compressor A takes its supply of air from the atmosphere and discharges the compressed air into the cooling coil B, where temperature and volume are both decreased at constant pressure. The cold air now passes into C, and in expanding helps to operate the compressor. The expanded air is delivered at low temperature and atmospheric pressure to the refrigerator-room, and as it passes through D its temperature increases and reaches that of D by the time it leaves at the right. The cycle of the air is completed by warming to the initial atmospheric temperature outside of the refrigerator.

The expansion in C being used to attain low temperature instead of work, care is taken not to heat the air during expansion, so that the expansion may be as nearly adiabatic as possible. In Fig. 43, let *anb* represent the passage of the air through a two-stage compressor A and the cooling tank B. During the expansion bc, in the working cylinder C, the temperature of the air drops below that maintained in the refrigerator T_r, the air being delivered at pressure p_a. As the temperature of the air increases along the constant-pressure curve ca, it extracts from the refrigerator the heat under the curve cd, in the $T\phi$-plane. The heat under *da, necessary to* complete the cycle, is obtained from

the atmosphere. That is, the refrigerating effect *cdef* is attained by the expenditure of the work *abcda*. If the compressor has but one stage, *amb*, the efficiency will be correspondingly less.

These machines are not economical, chiefly because the specific heat of air is so small that large quantities

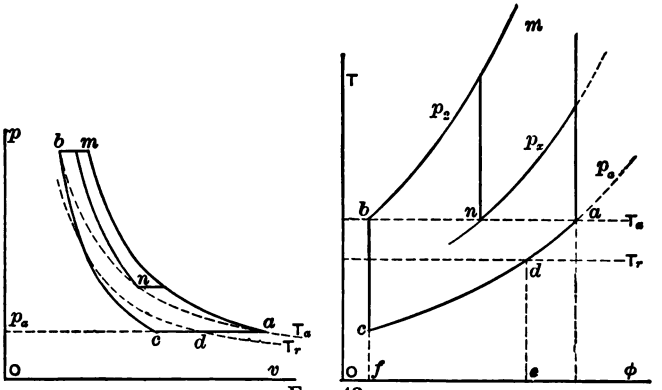

FIG. 43.

must be compressed to effect the desired refrigeration, thus necessitating large machines with large friction losses. Such plants are still used in places where it is more essential to guard against danger arising from the leakage of fluids, such as ammonia, than to install the most economical plant, as, for example, on war vessels.

Ammonia Refrigerating Plant.—A complete cycle of the working fluid in the second class of refrigerating *plants, where a* saturated vapor is used, differs mate-

rially from the above, as the substance is condensed and vaporized during the process. The more commonly used fluids are ammonia, carbon dioxide, and sulphur dioxide; ammonia being used most generally.

Fig. 44 shows diagrammatically the essential features of an ammonia refrigerating plant. It consists of (1) a

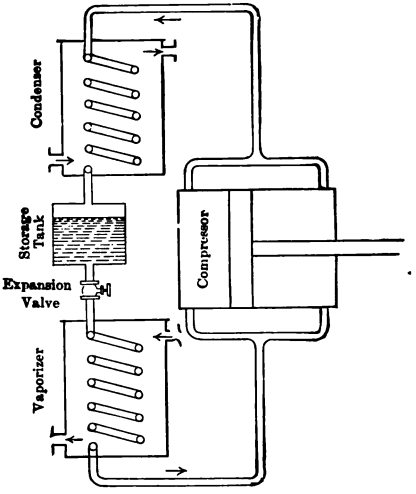

FIG. 44.

compressor which takes the low-pressure vapor from the refrigerator coils and delivers it at some higher pressure, (2) a condenser consisting of a series of coils in which the hot gas is cooled until it liquefies, (3) a storage-tank containing a supply of ammonia which remains liquefied under the high pressure at atmospheric temperature, (4) an expansion-valve from which the liquid emerges under reduced pressure, and (5) the

refrigerator coils in which the liquid, under reduced pressure, is vaporized by withdrawing the necessary heat from its surroundings. The high pressure prevails from the delivery-valve of the compressor to the expansion-valve, and low pressure from the lower side of the reducing-valve to the admission-valves of the compressor.

The refrigerator coils may be used directly, thus bringing the temperature of the surroundings down near the boiling temperature of the liquid, or, if such a low temperature is not desired, the coils may pass through a bath, as of brine, and reduce this to the desired temperature. The cold brine is then circulated through the refrigerator. This latter method gives a more nearly constant temperature. The least movement of the expansion-valve causes variations in the back pressure, and hence in the boiling temperature of the ammonia, which would affect the surrounding air if used directly, but which would be absorbed by the large heat capacity of the brine. The direct system is, however, simpler and less expensive to install and to maintain.

As long as any liquid remains unvaporized in the refrigerator coils these will remain at the temperature of vaporization, but afterwards the pipes assume the higher temperature of the bath or surrounding atmosphere, and then begin to superheat the vapor at constant pressure. This superheating is still further in-

creased in the pipe leading back to the compressor. After leaving the compressor the highly superheated vapor passes to the condenser, losing part of its superheat at constant pressure on the way. This process is finished in the condenser, and then the vapor begins to liquefy at the temperature corresponding to the high pressure. The liquid finally emerges cooled to the temperature of the cooling water and collects in the storage-tank above the expansion-valve ready for a new cycle.

The corresponding pv- and $T\phi$-cycles are shown in

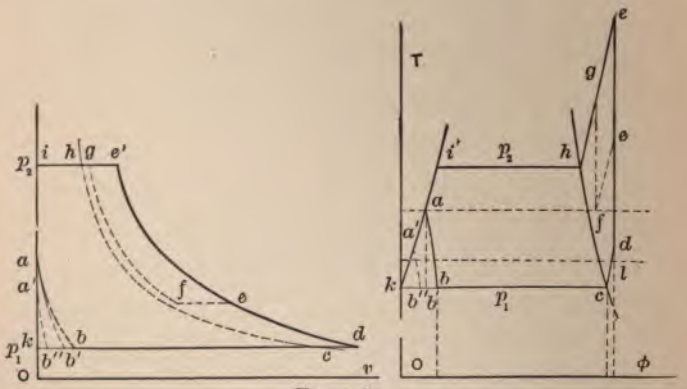

FIG. 45.

Fig. 45. ab represents the passage of the liquid through the expansion-valve along a constant-heat curve, during which a portion of the liquid $\frac{kb}{kc}$ is vaporized and *the refrigerative* power of its liquefaction destroyed;

bc represents the vaporization of the remainder of the liquid; *cd*, the superheating of the vapor during the last part of the refrigerator coils and the return pipe to the compressor; *de'* represents the compression, and *e'h*, the loss of superheat by conduction, radiation, etc., as the hot gas flows along the pipe to the condenser. If the compressor were two-stage, with intermediate cooling down to atmospheric temperature, the path followed would be *defgh*. The liquefaction is represented by *hi*, and the further cooling of the liquid down to atmospheric temperature by *ia*.

To decrease the work required to compress the gas, attempts are made in various types of compressors to cool it during compression by the use of water-jackets or by the direct injection into the cylinder of either liquid ammonia or oil. In such cases the compression is no longer adiabatic, but of the form $pv^n = p_1 v_1^n$, where n will depend in any given case upon the amount of heat extracted by the jackets or absorbed by the injected fluid.

The temperature of the entering vapor is usually considerably lower than that of the cooling water, so that heat is radiated only during the latter part of the stroke, when the temperature of the vapor greatly exceeds that of the water. The compression line of indicator-cards from such compressors should therefore approximate closely the adiabatic curve drawn through the commencement of the compression stroke.

and should begin to fall below it more and more only as the discharge pressure is approached.

If oil of the same temperature as the entering vapor is injected into the cylinder, it can affect the temperature only by absorbing heat as the gas is compressed. The specific heat of the oil is greater than that of the cylinder walls, and possibly conduction occurs somewhat more rapidly from vapor to oil and then oil to metal than it would directly from vapor to metal, especially if the oil is in a finely divided state. This can only result in changing slightly the exponent n of the compression curve $pv^n = p_1 v_1^n$.

The effect of injecting liquid ammonia is difficult to describe in general terms, as the results will differ according to the quantity injected and the various temperatures of the liquid, vapor, and cylinder walls. If the cylinder walls are assumed to be non-conducting and the injected liquid is previously cooled to the temperature of the refrigerator, the superheated vapor will then lose its superheat and in so doing suffer a drop in pressure, as the cylinder volume is momentarily constant. At the same time, however, the injected liquid will be partially vaporized, increased in volume, and thus effect an increase in pressure. The resultant effect in this case would undoubtedly be a net reduction of pressure and thus decrease the work of compression. As, however, the cylinder walls are *good conductors* when in contact with a liquid, enough

of the ammonia might thus be vaporized to produce a net increase in pressure. If the liquid is injected at the same temperature as the entering vapors, it is at a higher temperature than that of saturated vapor at the prevailing pressure, and hence will partially vaporize until a condition of equilibrium is established. What the final conditions of pressure and temperature will be will evidently depend upon the relative weights of liquid and vapor, the pressure, and the temperatures of vapor, liquid, and cylinder. In either of the above assumptions, if the resultant pressure is less and part of the liquid still remains unevaporated, the work of compression would be still further decreased, as saturated vapors transmit heat to the cylinder walls more rapidly than superheated vapors.

If the liquid is injected into the suction-pipe of the compressor, it will expand at the prevailing back pressure and reduce the temperature down to that corresponding to that pressure. Whether or not there results a net diminution in volume must depend upon the amount injected and the quantity of heat received from external sources. Although the work may or may not be decreased, according to circumstances, the temperature will at least be decreased, and thus the amount of necessary cooling water diminished.

It is theoretically possible to effect a further saving in liquid refrigerating plants by changing from the throttling-curve *ab* (Fig. 45) to a frictionless adiabatic

expansion ab'; that is, by replacing the expansion-valve with an auxiliary cylinder and thus utilize the expansive force of the ammonia to help run the compressor. The refrigerative power of the ammonia would be increased at the same time, since the amount vaporized during expansion would be decreased from kb to kb'. It is possible that the mechanical complications thus introduced would more than counterbalance the thermodynamic savings.

It has also been suggested that the loss in refrigerative power occasioned by the expansion ab could be diminished by reducing the temperature of the liquid at the point a. Thus the gas in passing from the refrigerator to the compressor absorbs from the atmosphere the heat represented by the area under ld. If such a loss is unavoidable, it could be neutralized by jacketing the return pipe with the liquid ammonia about to be fed to the refrigerator, thus reducing the temperature of the latter from a to a_1 and so decreasing the amount vaporized by expansion from kb to kb''.

The Warming-engine.—The third possibility of the reversed cycle, viz., the utilization of the heat delivered at the upper temperature for heating, was pointed out by Lord Kelvin; the idea being that given, say, a definite quantity of steam for heating purposes a greater heating effect could be obtained by utilizing *the steam to run* an engine and compressor system

and then diverting the exhaust-steam of the engine and the heated fluid of the compressor to heating purposes than by a direct application of the steam itself.

The explanation of this fact is that in the one case the availability of the heat to perform work at the high temperature is utilized, while in the other it is lost. When heat is transferred by conduction, radiation, etc., from a hot body to a colder body, the entropy of the hot body decreases and that of the cold body increases. As the decrease, $\frac{\Delta Q}{T_1}$, is less than the increase, $\frac{\Delta Q}{T_2}$, it follows that there is a net increase in the entropy of the system equal to $\Delta Q\left[\frac{1}{T_2}-\frac{1}{T_1}\right]$.

With the given quantities of heat Q_1 and Q_2 in the hot and cold bodies initially, the resultant uniform temperature T_3 will depend upon the entropy of the system, since $Q_1+Q_2=T_3\cdot\phi_3$. The smaller ϕ_3 the greater will be the value of T_3 for a given quantity of the cold body; or, if T_3 is a fixed temperature to which the cold body is to be raised, the smaller the entropy of the cold body the greater the quantity which could be raised to this temperature.

Hence any means of reducing the increase of entropy $\Delta Q\left[\frac{1}{T_2}-\frac{1}{T_1}\right]$ will increase the number of pounds of the cold body which can be raised to the desired temperature. The maximum value is evidently attained when

$JQ\left[\frac{1}{T_2}-\frac{1}{T_1}\right]=0$. This is possible in two ways: (1) isothermal transfer, during which the entropy of one part of the system decreases as fast as that of the remainder increases; or (2) by isentropic transfer changing heat into work and then back to heat again, by which means the entropy of each part of the system remains unchanged and the temperature of the hot body is decreased while that of the cold body is raised by quantities inversely proportional to the entropies of the two bodies. Evidently only the second form is applicable here. Let us discuss the ideal case where radiation losses do not exist.

Let T_1 be the temperature of a limited supply of heat, Q_1, and T_2 that of an unlimited supply, say, of the atmosphere, and T_3 some intermediate temperature to which a room is to be warmed. A Carnot engine working between T_1 and T_3 would perform the work $Q_1\frac{T_1-T_3}{T_1}$ and reject the heat $Q_1\frac{T_3}{T_1}$. Suppose this work to be expended upon an air-compressor, heating the air from T_2 to T_3. As the entropy of the air does not increase during adiabatic compression, the entropy of the air so compressed must be equal to the heat added during compression divided by the increase in temperature, or

$$\phi_2=\frac{Q_1\frac{T_1-T_3}{T_1}}{T_3-T_2}=\frac{Q_1}{T_1}\cdot\frac{T_1-T_3}{T_3-T_2},$$

hence the supply of air taken into the compressor must have contained the heat

$$Q_2 = T_2 \cdot \phi_2 = Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}.$$

The heat contained in the hot air at temperature T_3 will then be

$$Q_A = Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2} + Q_1 \frac{T_1 - T_3}{T_1}$$
$$= Q_1 \frac{T_1 - T_3}{T_1} \left[\frac{T_2 + T_3 - T_2}{T_3 - T_2} \right] = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2},$$

which could also be obtained by multiplying the uniform entropy of the air by the final temperature T_3. Thus

$$Q_A = \phi_2 \cdot T_3 = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}.$$

The total quantity of heat delivered to the room thus becomes the sum of that rejected by both engine and compressor, or

$$Q_3 = Q_1 \frac{T_3}{T_1} + Q_A = Q_1 \frac{T_3}{T_1} + Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2}$$
$$= Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_2}{T_3 - T_2}.$$

The increase in heating power over that obtained by the use of the steam alone is thus

$$Q_3 - Q_1 = Q_1 \frac{T_3}{T_1} \cdot \frac{T_1 - T_2}{T_3 - T_2} - Q_1 = Q_1 \frac{T_2}{T_1} \cdot \frac{T_1 - T_3}{T_3 - T_2} = Q_2.$$

or equal to the heat contained in the air originally. An examination of this last formula shows that if the upper and lower temperatures, T_1 and T_2, are fixed, the gain will be greater the smaller the value of T_3, so that when the range $T_3 - T_2$ is small the gain may be many times the original quantity of heat. The gain $Q_3 - Q_1$ represents the extreme difference obtained by supposing in one case all the availability to be utilized and in the other that none of it is utilized, or that the hot body simply expanded along a constant heat-curve until T_3 is reached, thus suffering an increase of its own entropy. In practice the actual difference would be diminished from both sides, the maximum value of Q_3 being impossible to attain, due to radiation, conduction, and friction losses, and the minimum value Q_1 would always be exceeded, as the heat contained in the air would be partially utilized.

This maximum gain, $Q_3 - Q_1$, can be illustrated by means of the $T\phi$-diagram, as shown in Fig. 46. Let *ab* represent the quantity of heat Q_1 at the temperature T_1. T_2 is the temperature of the atmosphere and T_3 that in the room. A Carnot engine working between T_1 and T_3 would perform the work *ac* and exhaust the heat *db*, the temperature of the exhaust having been lowered from T_1 to T_3, but the entropy remaining constant at *a'b*. If no work is performed, but the hot body *permitted to* expand along the constant heat-curve *H*, *the heating effect* at T_3 will be $de = ab = Q_1$. Let us rep-

resent the change in the condition of the air at the left of the line aa'. Let $mn(=ac)$ represent the work of the compressor in heat-units. As this has to bridge

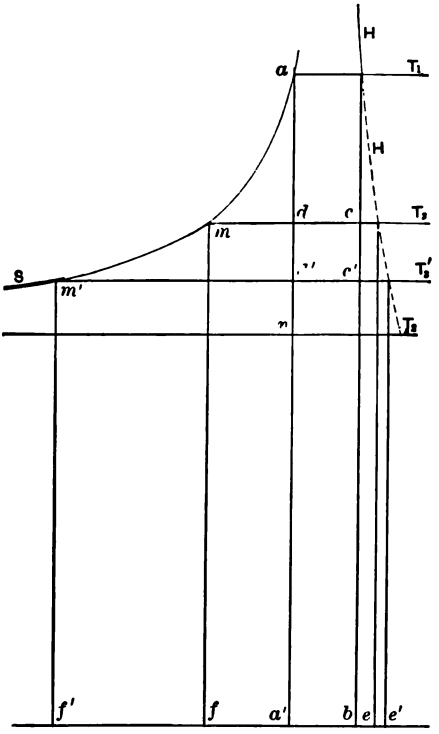

FIG. 46.

over the temperature interval $T_3 - T_2$, its width or entropy will be $fa' = \frac{ac}{T_3 - T_2}$. The heat originally contained in the air thus compressed is therefore shown by nf. The total heating effect is thus $mf + db$, which is greater than de by the area nf.

To determine the position of the point m, it is convenient to construct the rectangular hyperbola as passing through a, and so proportioned that when intersected by isothermals T_3, T_3', etc., the rectangles mn, $m'n$, etc., thus determined will be equal to the work performed by the Carnot engine, ac, ac', etc., respectively.

It should be noticed that as T_3 approaches T_2, the quantity of heat nf, nf', etc., utilized from the atmosphere increases indefinitely. "When, therefore," to quote the words of Prof. Cotterill, "we warm our houses by the direct action of heat from combustible bodies, we waste by far the greater part of it by making no use of the high temperature at which the heat is generated, a small quantity of heat at high temperature being ideally capable of raising a large quantity to a moderate temperature."

"It is interesting, and may some day be useful," says Prof. Ewing, "to recognize that even the most economical of the usual methods employed to heat buildings, with all their advantages in respect of simplicity and absence of mechanism, are in the thermodynamic sense spendthrift modes of treating fuel."

CHAPTER XII.

THE TEMPERATURE-ENTROPY DIAGRAM OF THE ACTUAL STEAM-ENGINE CYCLE.

THE Rankine cycle is based upon the following assumptions:

(1) Non-conducting cylinder walls and piston;

(2) Isentropic expansion to the back pressure;

(3) Instantaneous action of the valves;

(4) No leakage by the piston and the valves.

From the first two conditions it follows that the size of the clearance space is immaterial.

Referring to the actual steam-engine, we find that the conductivity of the metal produces initial condensation and reevaporation losses, and that the expansion can be carried to back pressure only by reducing the efficiency. The size of the clearance must therefore be considered, because the cycle of the clearance steam will affect the economy. The valves are not instantaneous in action, and leakage always occurs by both piston and valves. The Rankine cycle is thus unattainable in practice and is but an ideal which the actual engine strives to approximate.

The amount of condensation and reevaporation is the result of so many factors, that to determine the influence of each by the ordinary methods of comparison would require too much time and money. Hence to aid in evolving a theory of the steam-engine which shall account for all heat losses and interchanges, some convenient form of analysis must be adopted by which the losses for any single test may be investigated.

Hirn's analysis makes possible the determination of the net heat changes occurring between admission and cut-off, cut-off and release, release and compression, and compression and admission, but does not give information as to the actual direction of heat-transference at any moment. Fortunately, the $T\phi$-diagram offers a graphical solution equivalent to that of Hirn's analysis, and also makes clear the direction in which the interchange of heat is occurring at any point. Before a $T\phi$-projection of an indicator-card can be made, it will be necessary to discuss at length the different lines of the card in order to determine exactly what each represents.

The Admission Line of the Indicator-card.—During admission the steam is not at a uniform temperature and pressure. Part is still in the steam-pipe under boiler pressure, part has passed through the valve-chest and steam-ports, and has already entered the cylinder, *and still a third* portion is in the process of transition. *In general, the* surrounding metal is colder than the

steam, so that a continual loss of heat is experienced resulting in condensation and decrease of volume and entropy. To this is added the further effect of wire-drawing, due to too small steam passages and to the throttling effect when the valve is opening and closing, producing a drop of pressure and increase of both specific volume and entropy. It is probable that each particle of steam follows its own path in passing from the steam-pipe into the cylinder up to the point of cut-off. Thus the admission line of the indicator-card is not the pv-history of the entire quantity of steam nor of any particular part of it, and is only a record of the pressure exerted from moment to moment by the varying quantity of steam confined in the cylinder. Hence in projecting the admission-curve into the $T\phi$-plane it must be remembered that the projection does not represent the $T\phi$-history of any portion of the steam, but is simply a reproduction of each individual point of the pv-curve.

Let $a'c'$ (Fig. 47) represent the $T\phi$-projection of the admission line of an indicator-card, while b represents the state point of the steam in the steam-pipe. But for the various losses the admission line would have been ab, which represents the actual path followed by the steam in the boiler. If, for a moment, we consider the admission to represent a reversible process, the area under $a'c'$ will represent the heat received during *this process*. Hence the area $abb_1c_1c'a'$ represents the

difference between the heat contained per pound of steam in the boiler, and the amount realized per pound in the cylinder, or the losses due to initial condensation and wire-drawing. The former would simply result in the condensation of part of the steam, thus causing the value of x to diminish from b to c, or possibly to some point slightly to the left of c; the latter would cause a

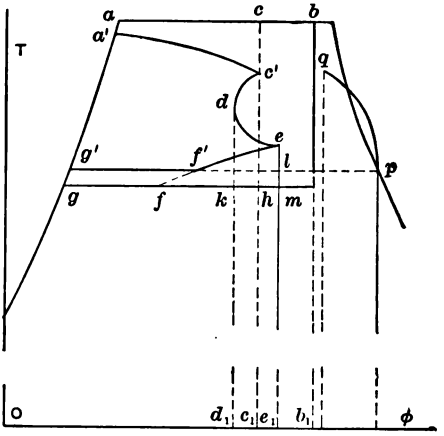

FIG. 47.

drop in pressure and increase of entropy, moving the state point to c'. Due to the impossibility of distinguishing accurately between these two opposing factors, one tending to decrease, the other to increase the entropy, the area cbb_1c_1 is taken to represent the loss due to initial condensation, and the area $acc'a'$ to represent the loss or reduction in efficiency due to friction and wire-drawing.

The Expansion Line of the Indicator-card.—If we assume that the leakage by piston and valves is negligible during expansion, the expansion-curve between cut-off and release represents the continuous pv-history of the entire quantity of steam contained in the cylinder; that is, of the cylinder feed plus the clearance steam. The temperature of the steam throughout the cylinder is not uniform, as heat-conduction is occurring between the steam and the metal, so that the indicator records but the average pressure due to these variable temperatures. Hence the $T\phi$-projection will give but average values of the $T\phi$-changes during expansion.

Since there is no appreciable friction of the steam against the metal, as during admission, it follows that neglecting the inequalities in the temperature of the steam, there is no reduction of the heat efficiency of the steam due to *internal* irreversible processes, and thus any increase or decrease of the entropy of the steam must result from heat-transferences between the steam and the surrounding metal. If adiabatic, the curve would here be isentropic, but as the steam is at first hotter than the cylinder walls, the flow of heat is from steam to metal, thus causing an increase in the entropy of the metal and a decrease in that of the steam.

The expansion line thus assumes at the start some such form as $c'd$, becoming steeper as the temperature drops, and just at the moment the temperatures of the

steam and the walls are the same it becomes isentropic. From this point, d, on to release at e, the heat transfer is from metal to steam, so that the entropy of the latter now increases and the curve slants to the right.

The ideal engine, supplied with steam of condition c', would expand isentropically along $c'c_1$ to the back pressure at h. Hence the area $c'dd_1c_1$, under the first part of the expansion-curve, represents the loss of heat due to conduction. Again, the ideal engine, supplied with steam of condition d, would expand isentropically to k. The area under de, for the actual engine, thus represents a gain due to the heat returned by the walls. It should be noted that the heat thus regained is restored at a lower temperature than that at which it was lost, and hence at a lower efficiency.

The Exhaust Line of the Indicator-card.—Let us consider first the case where the expansion is carried down to back pressure. The ideal engine, supplied with steam of quality e, would expand along ee_1 down to the pressure in the condenser and then condense along mg. The actual engine, due to the resistance of the exhaust ports, etc., would expand to some pressure, as l, greater than that in the condenser and would then exhaust along lg'. The area $lg'gm$ would thus represent the loss of heat due to throttling during exhaust.

If the release-valve opens at e before back pressure is reached, the phenomena are as follows: As the valve starts to open steam begins to escape and is throttled

down to the condenser pressure; as the valve continues to open the escape becomes more rapid until the back pressure is established. Then on the return stroke more steam is forced out against the back pressure, and near the end, as the exhaust-valve begins to close, there is a slight rise in pressure and a small quantity escapes, suffering reduction in efficiency by throttling. As the valve closes, all the cylinder feed has escaped, and only the clearance steam remains. It is necessary to note that the exhaust line of the card records the pressure of the steam still in the cylinder at any moment and gives no information whatever as to its condition, or of the condition of that portion already exhausted. Thus part of the steam has already reached the condenser (or in the case of a multiple-expansion engine the following cylinder or intermediate receiver), and has already parted with some of its heat, while that still in the cylinder, being at a lower temperature than the cylinder walls, is receiving heat and losing its moisture and may sometimes at compression have become even superheated. The last part of the exhaust-steam will necessarily have to retrace part of this thermodynamic process on reaching the condenser, or upon mingling with the rest of the steam in the following receiver or cylinder.

The exhaust line of the card does not represent the pv-history of any definite quantity of steam, but is simply a pressure record of continually varying quantities

confined in a constantly diminishing volume. It does, however, represent the amount of work required to discharge the steam, and in that sense the area under its $T\phi$-projection will represent the total heat discharged.

In the case of the ideal engine, the exhaust line, *efg*, divides into two parts, *ef* and *fg*, equivalent to decrease of pressure at constant volume and to decrease of volume at constant pressure respectively. The heat rejected is represented by the total area under *efg*, and exceeds that rejected after complete expansion to the back pressure by *efm*, which thus represents the extra loss incurred by incomplete expansion. The exhaust line for the actual indicator-card will be some such curve as $ef'g'$, where the area $ef'l$ shows the loss due to incomplete expansion, and $lg'gm$ the loss due to throttling, friction, etc.

The Compression Line of the Indicator-card.— The compression-curve, from the closing of the exhaust-valve up to the moment of admission, gives the *pv*-history of the clearance steam, and, if no leakage is assumed, the $T\phi$-projection will thus be the actual $T\phi$-history of a definite quantity of steam. As the pressure increases the curve deviates more and more rapidly from the adiabatic, due to the increasing effect of conduction losses, and on some cards may become *nearly* isothermal. In such cases it is probable that *the assumption* of dry steam at compression is incor-

rect, the presence of moisture helping to explain the rapid loss of heat.

During the interval between the opening of the admission-valve and the attainment of initial pressure the time interval is so small that probably the assumption of adiabatic compression of the clearance steam would not be greatly wrong. The gain in heat thus incurred must be at once lost by condensation during the first part of the admission, but it is impossible to determine the history of this change.

To obtain the $T\phi$-projection of the compression-curve, the saturation-curve for the weight of clearance steam should be drawn through the point of compression (assuming dry steam at compression) and the projection performed as previously described. The curve will assume some such form as pq, which may or may not, according to circumstances, have its course partially or wholly in the saturated or superheated regions. In any case the area under the curve, when reduced to the proper ratio, shows the heat lost to the walls during compression, and, if the horizontal line of the indicator-card is established at q, gives a general idea of the temperature of the cylinder at the moment of admission, and hence a measure of the heat necessary to bring the cylinder up to the temperature of the entering steam. Except for this one feature, the cycle of the clearance steam is unimportant, as all *the losses* occasioned by it will be manifested by the

difference between the cycle for an ideal engine, working between the given pressures in the steam pipe and condenser, and the $T\phi$-plot of the actual card.

The Indicator-card.—Considered as a whole, the indicator-card furnishes the following information. The admission line and the exhaust line simply represent the pressure of part of the steam, but do not give any information regarding the specific volume. On the other hand, both the expansion and the compression lines give the history of definite quantities of steam. Thus both the expansion and compression of the clearance steam are recorded, while only the expansion of the cylinder feed appears, the compression of the latter occurring in the boiler. The entire card, if plotted directly, can at the best be considered only as the heat equivalent of the work done, but not as the $T\phi$-history of any closed cycle. Thus, for example, while the projection of the exhaust line gives some such curve as $ef'g'$, which on the $T\phi$-chart indicates condensation, it is probable that the value of x of the confined steam is actually increasing.

The difficulties involved in the proper interpretation of the irreversible portions of the indicator-card have led different investigators to make certain assumptions as to the influence of the clearance steam and as to the *possibility of replacing* the actual curves by equivalent *reversible processes.*

The Clearance Steam Considered as an Elastic Cushion. —During expansion the clearance steam follows the same laws and variations as the cylinder feed, and this in general is not the reverse of its history during compression. Thus the cycle of the clearance steam, if it could be drawn, would enclose an area representing either positive or negative work. This cycle would then be of especial interest in determining losses experienced by the clearance steam, but as these losses must eventually be charged against the entering steam, the total effect upon the efficiency would be the same if the clearance steam were considered as an isolated elastic cushion expanded and compressed along the same adiabatic. If, then, an adiabatic is drawn through the point of compression on the indicator-card, the horizontal distance from any point on this adiabatic to the corresponding point on the indicator-card shows the volume which the cylinder feed would occupy under the above assumptions. Taking this adiabatic as the line of zero volume, a diagram can thus be constructed which shows only the variations of the cylinder feed. It is then only necessary to draw on the saturation-curve for the weight of steam fed to the cylinder per stroke and the card can be at once projected into the $T\phi$-plane. This is, in its essential points, the method adopted by Prof. Reeve in his book on the *Thermodynamics of Heat Engines*, although he changes *volumes so* that the reconstructed card represents

the pv-history of one pound of cylinder feed instead of the actual weight in the cylinder. To project his reconstructed card into the $T\phi$-plane, it is only necessary to draw the saturation-curve for one pound of steam instead of that for the pounds fed per stroke. Inasmuch as the $T\phi$-projection will be the same in either case it seems somewhat simpler to adopt the first method, viz., to construct the saturation-curve for the number of pounds in the cylinder rather than to redraw the diagram to correspond to one pound of cylinder feed.

This method undoubtedly makes possible a determination of the general magnitude and character of the various heat interchanges, but is open to the following objections.

The compression line of the card refers to the clearance steam alone, so that the deviations from the adiabatic thus obtained refer to itself and not to the cylinder feed. Furthermore, the reconstructed curve may actually pass to the left of the water line, assuming imaginary values on the $T\phi$-plane, and thus give a wrong conception of the condition of this steam, which, instead of being wet, is usually dry or superheated, and lies to the right of the expansion line. Again, the expansion line no longer represents the actual pv- or $T\phi$-history of the steam, but an imaginary history which *the* cylinder feed might have if the clearance steam *expanded adiabatically*. The entire card thus becomes

to a certain extent imaginary, and in so far is undesirable.

The Indicator-card Considered as a Reversible Cycle. —The area of the card gives the heat changed into work, and this same result may be attained by assuming the clearance steam and cylinder feed to remain in the cylinder, being heated and cooled by external means and thus caused to expand and contract along a reversible cycle coincident in shape with the actual card. The original card may thus be projected into the $T\phi$-plane as soon as the saturation-curve for the total weight of steam has been drawn on it. The expansion line represents the actual history of the substance, but the compression line is entirely imaginary.

This method was adopted by Prof. Boulvin in his book, *The Entropy Diagram*, from which the following two illustrations are copied, with but slight alterations. *ABCDEF*, in Fig. 48, represents the $T\phi$-projection of a certain indicator-card. The line *DE* represents the actual expansion history of the total steam; the other lines give more or less imaginary values. The small diagram at the right is used to interpret the compression line *AB*. If W and w represent the pounds of cylinder feed and clearance steam respectively per revolution, the large diagram represents the cycle of $W+w$ pounds of the mixture, while the small diagram represents the entropy of w pounds. Assuming dry steam at compression, the heat rejected by w pounds

during compression is shown by area abb_1a_1. The entropy $a_0a = A_0A$ is that due to the vaporization of w pounds of water, so that AG must represent that for the W pounds of cylinder feed. Through A and G it is then possible to draw the water line AL and the dry-steam line GX for W pounds. The cycle for the

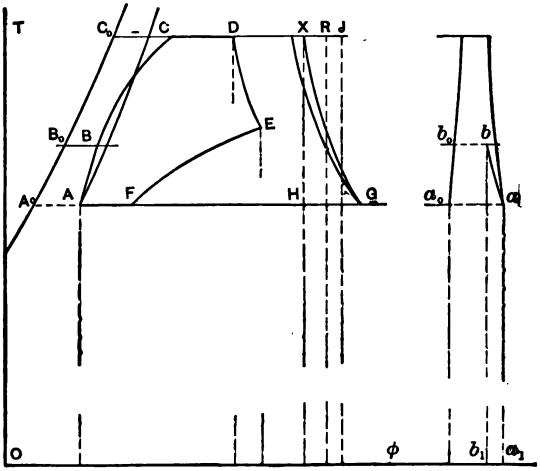

FIG. 48.

Rankine engine using W pounds per revolution is represented by $ALXH$, so that the efficiency of the actual engine as compared with that of the ideal engine is

$$\frac{ABCDEF}{ALXH}.$$

In Fig. 49, let L' and L be the liquid lines of W and $W+w$ *pounds* respectively, and let L'' be drawn through

the point of compression a parallel to L'; and let S and S' be the dry-steam lines corresponding to the water lines L and L''. As is evident from Fig. 48, the horizontal distance between the liquid line A_0C_0 and the compression line AB is equal to the entropy of vaporization of the clearance steam (for example, $A_0A = a_0a$, $B_0B = b_0b$). In Fig. 49 the horizontal distance between L' and L represents the entropy for $W+w-W$ or w

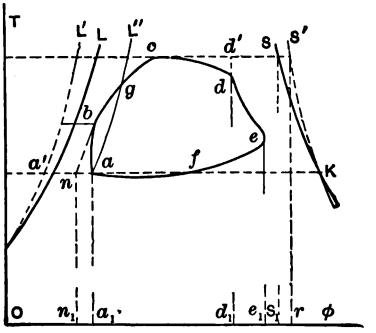

FIG. 49.

pounds of water. Therefore the horizontal distance from L' to the compression-curve ab represents the total entropy of the clearance steam. $a'L'$ is thus the curve of zero entropy for the clearance steam, and any curves, such as nb and aL'', parallel to $a'L'$ represent isentropic changes. The heat rejected along ab is thus shown by $abnn_1a_1$ without the help of any auxiliary diagram.

$a_1aL''S'r$ represents the heat received per cycle in the

W pounds of cylinder feed, *abcdef* represents the heat utilized, so that the difference, M, represents the total heat losses. These consist of the exhaust-heat a_1aee_1, the heat lost during compression $abnn_1a_1$, and the heat lost during admission. The heat refunded during expansion is represented by dee_1d_1. Hence the heat lost during admission equals

$$M - a_1aee_1 - abnn_1a_1 + dee_1d = cL''gcdd_1rS'c - n_1nbgaa_1.$$

If the condition of the steam is desired at any point of the expansion *de*, it is found by reference to the lines L and S and not with reference to L'' and S'. Hence the initial condensation changed the state point from S to d', so that this loss is represented by the area under Sd' and not by that under $S'd'$. Subtracting this from the total loss during admission there is left

$$cL''gcdd' + SS_1rS' - n_1nbgaa_1$$

as the heat losses due to wire-drawing and friction.

This method is simple and easy of application, as it requires the construction of only two extra curves, L' and S'. Furthermore it gives a very complete account of the clearance steam. The only objection, apparently, is that, with the exception of the expansion line *de*, it gives an entirely false idea of the cycle of the cylinder feed, as this in reality is entirely condensed and *then heated along* aL'' in the boiler.

If the cylinder is jacketed, the heat given out by the jacket steam may be indicated at the right of XH (Fig. 48), as shown under XJ. Further, if the radiation loss is known, it may be represented under JR, and then the remaining area under XR will represent the heat given by the jackets to the steam in the cylinder.

Separate Cycles for the Cylinder Feed and the Clearance Steam.—Whatever the assumptions made with reference to the card, its total area must not be changed. Thus, even if an attempt is made to draw separate cycles for both cylinder feed and clearance steam, the compression line of the resultant cylinder-feed card will never exactly coincide with the water line of the $T\phi$-diagram, so that this line must always remain imaginary in its readings. It is, however, possible to have the reconstructed expansion line represent the true pv-history of the steam. Thus, in place of drawing an isentropic line through the point of compression, draw the polytropic curve $pv^n = C$, where n has the value found for the expansion-curve. Assuming that the clearance steam is expanded and compressed along this line, the card for the cylinder feed can be constructed by assuming this curve to represent zero volume. The $T\phi$-projection of the expansion-curve will thus represent the true average history of the cylinder feed, and the compression line will follow more closely the water line than it does when an adiabatic curve is taken as *the new* base line.

The cycle for the clearance steam can be found between cut-off and release, and between compression and admission, but the rest of it would be entirely imaginary. Possibly it might be continued from admission up to the attainment of initial pressure by assuming adiabatic compression. As may be seen from Fig. 47, the clearance steam contains less moisture at admission and compression than at cut-off and release respectively, so that whatever its exact path between admission and cut-off and between release and compression, it must at least show decreasing and increasing values of X respectively, so that its history is the reverse of that shown by the indicator-card itself.

It is doubtful if the increased labor involved in the making of such a plot would be recompensed by any added information which could not be obtained by a proper interpretation of the simple method used by Prof. Boulvin.

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